Enhanced Identification of Battery Models for Real-Time Battery Management

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Abstract—Renewable energy generation, vehicle electrification, and smart grids rely critically on energy storage devices for enhancement of operations, reliability, and efficiency. Battery systems consist of many battery cells, which have different characteristics even when they are new, and change with time and operating conditions due to a variety of factors such as aging, operational conditions, and chemical property variations. Their effective management requires high fidelity models. This paper aims to develop identification algorithms that capture individualized characteristics of each battery cell and produce updated models in real time. It is shown that typical battery models may not be identifiable, unique battery model features require modified input/output expressions, and standard least-squares methods will encounter identification bias. This paper devises modified model structures and identification algorithms to resolve these issues. System identifiability, algorithm convergence, identification bias, and bias correction mechanisms are rigorously established. A typical battery model structure is used to illustrate utilities of the methods.

Index Terms—Battery management system, battery model, parameter estimation, identifiability, system identification, bias correction, convergence.

I. INTRODUCTION

RENEWABLE energy generation, vehicle electrification, and smart grids rely critically on energy storage devices for enhancement of operations, reliability, and efficiency. A large-scale battery system consists of hundreds even thousands of battery cells, which have different characteristics even when they are new, and change with time and operating conditions due to a variety of factors such as aging, operational conditions, and chemical property variations. State of charge, battery health, remaining life, charge and discharge resistance and capacitance demonstrate nonlinear and time-varying dynamics [4], [5], [8], [12], [18], [24]. Consequently, for enhanced battery management, reliable system diagnosis, and improved power efficiency, it is necessary to capture battery cell models in real time [18]. This paper aims to develop a real-time automated battery characterizer that will capture individualized characteristics of each battery cell and produce updated models in real time. The core of such a system is advanced system identification techniques [15], [29], [27], [28], [29] that provide fast tracking capability to update each battery cell’s individual model when it is plugged into the battery system and to track battery aging and health conditions during operation.

The main idea of such identification methods includes: (1) using load changes to “stimulate” or “excite” the internal behavior of the cell; (2) using measured voltage and current data to estimate model parameters such that the model-produced voltage and current trajectories match these from the actual battery cell; (3) employing recursive algorithms to accomplish automated model generation and real-time operation so that new measurement data are immediately used to update the model.

To facilitate discussions on the key issues and challenges in real-time battery model identification, we employ, as a typical model structure, the resistance-capacitance battery model, which is part of ADVISOR developed at National Renewable Energy Laboratory (NREL) [12]. The parameters of the components are functions of the state of charge (SOC) and cell temperature. In addition, the resistance values differ when the battery is in “charge” mode or “discharge” mode. Typically the parameters of the model and their dependence on the SOC and temperatures are experimentally established. In this work, we will use system identification methods to identify them in real time, without lab testing facilities. When identification can capture the circuit model parameters faster than a battery’s charge/discharge operations, one may employ the “frozen time” concept and view the model parameters as their linearized values at a given operating condition. As such identification of the system can concentrate on time-varying but linearized models.

The goal of system identification is to identify system parameters on the basis of observations on the measured inputs and outputs. Continuing our recent work on battery models [22], [23], in this paper we will use adaptive filtering algorithms to derive convergent algorithms. System identifiability, algorithm convergence, identification bias, and bias correction mechanisms are rigorously established. The paper is organized as follows. Section II introduces battery model structures. Although our methodology is generic, it is helpful to use a typical model structure to reveal key issues and discuss their resolutions. System identifiability is first discussed in Section
III. It is shown that physically meaningful models may not be real-time identifiable. Modifications of model structures to facilitate system identification are studied, leading to a modified regression expression for system identification. The section further explores the issue of identification bias correction. It is revealed that due to noise corruption in observation data and inherent correlation in the regressors, the standard least-squares (LS) algorithms will encounter identification bias, implying that parameter estimates will have an inherent error even when data sizes become very large. Bias correction measures are introduced to overcome this drawback. Strong convergence of the modified algorithms is established. For computational efficiency, recursive algorithms are derived. The methodologies of this paper are then evaluated and illustrated by using simulation studies in Section IV. Both standard LS algorithms and modified algorithms are illustrated for their utilities. Finally, our findings and potential applications are discussed in Section V.

II. Battery Model Structures

The methodologies developed in this paper can be applied to different battery models. We choose a well-developed and validated model structure to facilitate discussions of essential issues, including identification algorithms, identifiability, bias correction, noise correlation, and convergence analysis. We employ the resistance-capacitance battery model which is part of ADVISOR developed at National Renewable Energy Laboratory (NREL) [3], [12]; see Figure 1.

![Battery Model](image)

The model contains two capacitors ($C_b$ and $C_c$) and three resistors ($R_c$, $R_b$, and $R$). The capacitor $C_b$ models the main storage capacity of the battery. The capacitor $C_c$ captures the fast charge-discharge aspect of the battery and is much smaller than $C_b$. The parameters of the components are functions of the SOC and cell temperature ($T$). In addition, the resistance depends on also if the battery is in "charge" or "discharge" mode. These will be expressed as $R_b(S,T,\eta), R_c(S,T,\eta), R(S,T,\eta), C_b(S,T), C_c(S,T)$ when needed. To describe the model details, we define the following symbols.

- $v$ = terminal voltage (V)
- $i$ = terminal current (A)
- $v_b$ = voltage of the capacitor $C_b$ (V)
- $i_b$ = current through the capacitor $C_b$ (A)
- $v_c$ = voltage of the capacitor $C_c$ (V)
- $i_c$ = current through the capacitor $C_c$ (A)
- $T_a$ = air temperature (deg C)
- $T$ = cell temperature (deg C)
- $q$ = conducting heat transfer rate (W)
- $q_b$ = heat transfer rate generated by the battery cell (W)
- $q_{ac}$ = air conditioning forced heat transfer rate (W)
- $S$ = SOC
- $S_b = \text{SOC}_{C_b}$
- $S_c = \text{SOC}_{C_c}$
- $\eta = \begin{cases} 1, & \text{charge;} \\ 0, & \text{discharge} \end{cases}$

It was given in [12] that the overall state of charge is a weighted combination of the states of charge on $C_b$ and $C_c$

$$\text{SOC} = \alpha_b\text{SOC}_{C_b} + \alpha_c\text{SOC}_{C_c},$$

where $\alpha_b + \alpha_c = 1$. In the NREL/Saft model, $\alpha_b = 20/21$ and $\alpha_c = 1/21$. $\text{SOC}_{C_b}$ and $\text{SOC}_{C_c}$ will be related to the voltages on $C_b$ and $C_c$.

The thermal model is a lumped first order linear dynamics, shown in Figure 2.

$$q = \frac{T - T_a}{R_T}, \quad C_T\dot{T} = q_b - q - q_{ac},$$

where $R_T$ is the equivalent thermal resistance (C/W) and $C_T$ is the equivalent heat capacitance (J/C).

![Battery Thermal Model](image)

In [3], [12], the parameters of the model and their dependence on the SOC and temperatures are experimentally established. In this work, we will use system identification methods to identify them in real time, without lab testing facilities.

The states of charge of $C_b$ and $C_c$ are closely related to their voltages $v_b$ and $v_c$. The relationship $S_b = g_b(v_b)$ and $S_c = g_c(v_c)$ can be experimentally established. In the normal operating ranges of $S$ and $T$, they can be well approximated by linear functions [12].

From $S = \alpha_b g_b(v_b) + \alpha_c g_c(v_c)$, model parameters become functions of the state variables $v_b, v_c, T$ and the mode $\beta$

$$R_b = f_{R_b}(v_b, v_c, T, \eta), R_c = f_{R_c}(v_b, v_c, T, \eta), R = f_R(v_b, v_c, T, \eta),$$

$$C_b = f_{C_b}(v_b, v_c, T), C_c = f_{C_c}(v_b, v_c, T).$$

(2)
The battery model is derived from the following basic equations
\[
\begin{align*}
C_b v_b &= -i_b \\
C_c v_c &= -i_c \\
v_b - i_b R_b &= v_c - i_c R_c \\
i &= i_b + i_c \\
v &= v_c - i_c R_c - i R \\
q &= \frac{V}{R_c} \\
C_f T' &= q_b - q - q_{ac}
\end{align*}
\]

These lead to
\[
i_c = \frac{v_c - v_b}{R_c + R_b} + \frac{R_b}{R_c + R_b} i \\
i_b = \frac{v_b - v_c}{R_c + R_b} + \frac{R_c}{R_c + R_b} i
\]

With state variables \(v_b, v_c, T\); inputs \(i\) (current load), \(T_o\) (air temperature), \(q_b\) (battery cell generated heat flow rate), and \(q_{ac}\) (convective heat flow rate due to cooling air); operating mode \(\eta\); and outputs \(v\) (terminal voltage), \(S\) (state of charge), and \(T\); the state space model is
\[
\begin{align*}
\dot{v}_b &= \frac{v_b - v_c}{(R_c + R_b)C_b} + \frac{R_b}{(R_c + R_b)C_b} i \\
&= f_1(v_b, v_c, T, \eta) + g_1(v_b, v_c, T, \eta)i \\
\dot{v}_c &= \frac{v_b - v_c}{(R_c + R_b)C_c} + \frac{R_c}{(R_c + R_b)C_c} i \\
&= f_2(v_b, v_c, T, \eta) + g_2(v_b, v_c, T, \eta)i \\
\dot{T} &= \frac{1}{c_p} q_b - \frac{T}{c_p R_c} + \frac{T}{c_p R_c} - \frac{1}{c_p} q_{ac} \\
v &= \frac{R_b v_b - R_c v_c}{R_c + R_b} - \frac{R_c R_b}{R_c + R_b} + R) i \\
&= h_1(v_b, v_c, T, \eta) + m_1(v_b, v_c, T, \eta)i \\
S &= \alpha_b g_0(v_b) + \alpha_c g_c(v_c) \\
&= h_2(v_b, v_c, T) \\
T &= [0, 0, 1] \begin{bmatrix} v_b \\ v_c \\ T \end{bmatrix}
\end{align*}
\]

Remark 1: By (2), the model parameters depend on the state. Consequently, the system is highly nonlinear. Within the inputs, \(i\) is measured and fed back as an input to the model. \(T_o\) is a disturbance but is measured with some measurement noise. Since \(q_b\) is not modeled in detail, it will be considered as an unmeasured disturbance to the battery system. \(q_{ac}\) is controlled (by the cooling system). The cooling system dynamics is not considered in this model. As a result, we will view \(q_{ac}\) as a control input. The outputs \(v\) and \(T\) are measured with measurement noise. \(S\) is not measured.

Denote the state vector by \(x = [v_b, v_c, T]^{T}\), input vector \(u = [i, T_o, q_b, q_{ac}]^{T}\), output vector \(y = [v, S, T]^{T}\). The state space model (3) may be written in an abstract form, for methodology and algorithm development, as
\[
\begin{align*}
\dot{x} &= f(x, \eta) + g(x, \eta)u \\
y &= h(x, \eta) + m(x, \eta)u
\end{align*}
\]

This is a nonlinear system in an affine form. Since \(\eta\) is a control variable taking only two possible values, the system (4) is a hybrid system.

For simulation studies, we will use the NREL/Saft model [10], [11], [12]. The model structure is given in Figure 1. Nominal values of the parameters at temperature 20 (deg C) are
\[
\begin{align*}
C_b &= 82 kF, C_c = 4.074 kF, R_b = 1.1 m\Omega, \\
R_c &= 0.4 m\Omega, R = 1.2 m\Omega.
\end{align*}
\]

Although dependence of these parameters on \(S\) and \(T\) can be derived from experimental data, in this paper we employ system identification methods to capture real-time parameter values.

III. IDENTIFICATION OF BATTERY MODELS

Although system parameters in the battery model (3) can be established by using experimental data from lab testings, for real-time operation under a battery management system (BMS), the parameters must be derived using real-time operational data. This is due to several factors: (1) New batteries have different characteristics even for the same model, due to manufacturing variations. (2) Battery features depend on many factors that cannot be totally captured in model details. (3) Batteries experience significant aging effects. Consequently, it is not only desirable, but in fact imperative that model parameters be obtained individually in real time.

A. Identifiability of Battery Models

We first derive the transfer function for the battery model shown in Figure 1. Using \(1/C_s\) for the capacitors and deriving the transfer function in the s-domain, we obtain
\[
\begin{align*}
\frac{V(s)}{I(s)} &= \frac{d_1 s^2 + d_2 s + 1}{c_1 s^2 + c_2 s} \\
\end{align*}
\]

where \(d_1 = C_b C_c (R R_b + R R_c + R_b R_c), d_2 = R C_c + R C_b + R_c C_c + R_b C_b, c_1 = C_b C_c (R_b + R_c), c_2 = C_b + C_c.\)

For example, under \(C_b = 82 kF, C_c = 4.074 kF, R_b = 1.1 m\Omega, R_c = 0.4 m\Omega, R = 1.2 m\Omega,\) we have
\[
\begin{align*}
d_1 &= 748.31, d_2 = 195.12, c_1 = 501102, c_2 = 86074.
\end{align*}
\]

The input-output model (6) contains four coefficients. However, the internal circuit model in Figure 1 contains five parameters. In other words, there will be one redundant parameter in the model. As a result, the circuit model is not input-output identifiable when only the terminal voltage and current are measured.

Due to its physical meanings, the \((R_c, C_c)\) pair represents a much faster branch than the \((R_b, C_b)\) branch. Consequently, it is a sensible choice to let \(R_b = 0,\) reducing the battery system into a 4-parameter circuit model. We show now that this simplification results in a direct inverse mapping from the transfer function parameters to the circuit parameters.

When \(R_b = 0,\) (6) is reduced to
\[
\begin{align*}
\frac{V(s)}{I(s)} &= \frac{R C_b C_c R_b s^2 + (R C_c + C_b) R_0 C_b s + 1}{C_b C_c R_b s^2 + (C_b + C_c)s} \\
&= \frac{d_1 s^2 + d_2 s + 1}{c_1 s^2 + c_2 s}
\end{align*}
\]

where \(T_b = R_b C_b, d_1 = R C_T b, d_2 = R (C_c + C_b) + T_b, c_1 = C_c T_b, c_2 = C_b + C_c.\) The inverse mapping of the parameters can be derived as
\[
\begin{align*}
R &= \frac{d_1}{c_1}, T_b = d_2 = R (C_c + C_b) + T_b, c_1 = C_c T_b, c_2 = C_b + C_c, R_b = T_b / C_b.
\end{align*}
\]

This defines a mapping \(\beta = H_1(\mu)\) from \(\mu = [d_1, d_2, c_1, c_2]^{T}\) to \(\beta = [R, C_c, R_b, C_b]^{T}.\) As a result, simplified circuit model (8) is always identifiable.
B. Regression Structure for System Identification

Although the battery model’s physical input is the load (current $i$), since the model contains an integrator, for system identification the input/output setting needs to be modified.

Suppose that the signal sampling interval for a BMS is $\tau$. Let the sampled values be $v_k = v(k\tau)$, $i_k = i(k\tau)$, etc. To derive a sampled expression to identify parameters in (9), we note that

$$\frac{\dot{I}(s)}{V(s)} = Y(s) = \frac{c_1 s + c_2}{d_1 s^2 + d_2 s + 1} = \frac{\frac{c_1}{d_1} s + \frac{c_2}{d_1}}{s^2 + \frac{d_2}{d_1} s + \frac{1}{d_1}}$$

(10)

where $Y(s) = I(s)$. Using the forward approximation, we have $y_k = y_{k-1} + \tau i_{k-1}$. (10) is mapped to

$$G(z) = \frac{b_1 z + b_2}{(z-1)^2 + \frac{d_2}{d_1} z + \frac{1}{d_1}} = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

(11)

where $b_1 = \frac{\tau}{2} d_1$, $b_2 = \frac{\tau}{2} d_2^2 - \frac{d_1}{2} \tau$, $a_1 = \frac{d_1}{2} \tau - 2$, $a_2 = 1 - \frac{d_1}{2} \tau + \frac{1}{d_1}$. For the data from (7) and $\tau = 0.01$ second, we have

$$b_1 = 6.6964, b_2 = -6.6849, a_1 = -1.9974, a_2 = 0.9974$$

(12)

Define $\theta = [a_1, a_2, b_1, b_2]'$, $\mu = [d_1, d_2, c_1, c_2]'$ can be derived as a mapping $\mu = H_2(\theta)$ from

$$d_1 = \frac{\tau^2}{1 + a_1 + a_2}, d_2 = \frac{(2 + a_1) \tau}{1 + a_1 + a_2},$$

$$c_1 = \frac{b_1 \tau}{1 + a_1 + a_2}, c_2 = \frac{b_1 + b_2}{1 + a_1 + a_2}.$$  

Consequently, we have a mapping to the circuit parameters $\beta = H_1(\mu) = H_1(H_2(\theta)) = H(\theta)$.

(13)

The discrete-time system (11) leads to the regression expression of the new input/output relationship

$$y_k = -a_1 y_{k-1} - a_2 y_{k-2} + b_1 v_{k-1} + b_2 v_{k-2} = \phi_k' \theta$$

(14)

where the regressor is $\phi_k = \left[ -y_{k-1}, -y_{k-2}, v_{k-1}, v_{k-2} \right]$. It should be pointed out that in practical applications, the measurements on the current $i_k$ and voltage $v_k$ are subject to noises. As a result, $y_k$ is measured as $\tilde{y}_k = y_k + e_k$ and $v_k$ as $\tilde{v}_k = v_k + e_k$. Consequently,

$$\tilde{\phi}_k = \left[ -\tilde{y}_{k-1}, -\tilde{y}_{k-2}, \tilde{v}_{k-1}, \tilde{v}_{k-2} \right] = \left[ -y_{k-1} - \epsilon_{k-1}, -y_{k-2} - \epsilon_{k-2}, v_{k-1} + \epsilon_{k-1}, v_{k-2} + \epsilon_{k-2} \right] = \phi_k' + \delta_k$$

where $\delta_k = [\epsilon_{k-1}, -\epsilon_{k-2}, -\epsilon_{k-1}, \epsilon_{k-2}]$. In other words, noises enter the regressor also. This is a more complicated problem of errors-in-variable identification.

C. Identification Algorithms

For algorithm implementation, only $\tilde{y}_k$ and $\tilde{v}_k$ can be used. From the observation equation

$$\tilde{y}_k = y_k + e_k = \phi_k' \theta + e_k = (\tilde{\phi}_k' - \delta_k) \theta + e_k$$

and the mapping $\beta = H(\theta)$ we would like to derive identification algorithms to track time-varying parameters.

**Assumption I:** Suppose that the measurement on $y_k$ is subjected to measurement noise $\{e_k\}$ and the voltage measurement is subject to measurement noise $\{\epsilon_k\}$. The joint vector sequence $\{(e_k, \epsilon_k)\}$ is stationary and ergodic (in the sense of convergence with probability one (w.p.1)) such that $E(e_k', e_k') = 0$, $E((e_k', \epsilon_k')^2) < \infty$, and that both $\{(e_k', e_k')\}$ and $\{(\epsilon_k', e_k')\}$ are ergodic. That is,

$$\frac{1}{N} \sum_{k=1}^{N} (e_k, e_k') \rightarrow 0 \text{ w.p.1 as } N \rightarrow \infty,$$

$$\frac{1}{N} \sum_{k=1}^{N} (e_k, e_k') (e_k', e_k') \rightarrow Q \text{ a nonnegative definite matrix w.p.1 as } N \rightarrow \infty.$$

Note that we do not need the sequences $\{e_k\}$ and $\{e_k\}$ to be independent. A sufficient condition to ensure the ergodicity in the above assumption is that the underlying sequence is a stationary $\varphi$-mixing sequence, which is a sequence whose remote past and distant future are asymptotically independent. The well-known results [13, p. 488] then yield that $\{(e_k, e_k')\}$ and $\{(\epsilon_k', e_k')\}$ are strongly ergodic.

It is noted that for time-varying systems, identification of system parameters can only be performed on the basis of most recent data, such as a finite-time moving window or exponentially-decaying window. Hence, suppose the data on $i(t)$ and $v(t)$ are collected in a finite-time window $[0, T]$. Let $N = [T/\tau]$, the largest integer below $T/\tau$. When $\tau \rightarrow 0 $, $N \rightarrow \infty$, and convergence analysis in this case should be interpreted as asymptotic accuracy of identification when sampling intervals become small.

Let

$$\tilde{Y}_N = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix}, \quad Y_N = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad \tilde{\Phi}_N = \begin{bmatrix} \tilde{\phi}_1' \\ \tilde{\phi}_2' \\ \vdots \\ \tilde{\phi}_N' \end{bmatrix}, \quad \Phi_N = \begin{bmatrix} \phi_1' \\ \phi_2' \\ \vdots \\ \phi_N' \end{bmatrix}, \quad \Delta_N = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_N \end{bmatrix}.$$

(15)

Then, $\tilde{Y}_N = Y_N + E_N$; $\tilde{\Phi}_N = \Phi_N + \Delta_N$. From $\tilde{y}_k = (\tilde{\phi}_k' - \delta_k) \theta + e_k$, the observation equation becomes $\tilde{Y}_N = (\Phi_N - \Delta_N) \theta + E_N$. When $\tilde{\Phi}_N \Phi_N$ is nonsingular, w.p.1, the standard LS estimate is

$$\theta_N = (\tilde{\Phi}_N \Phi_N)^{-1} \tilde{\Phi}_N \tilde{Y}_N = (\tilde{\Phi}_N \Phi_N)^{-1} \tilde{\Phi}_N \tilde{Y}_N; \beta_N = H(\theta_N).$$

(16)

We should demonstrate that due to signal correlation, this algorithm will introduce identification bias, namely, parameter estimates will converge to values away from the true value.

D. Identification Bias and Correction

Under Assumption 1, since $Y_N$ and $\Phi_N$ are deterministic, as $N \rightarrow \infty$, with probability one,

$$\frac{1}{N} \Phi_N' Y_N \rightarrow A; \quad \frac{1}{N} \Phi_N' E_N \rightarrow 0; \quad \frac{1}{N} \Phi_N' \Phi_N \rightarrow R; \quad \frac{1}{N} \Delta_N' \Delta_N \rightarrow \Sigma; \quad \frac{1}{N} \Phi_N' \Delta_N \rightarrow 0,$$

which imply

$$\frac{1}{N} \tilde{\Phi}_N' \tilde{Y}_N \rightarrow A + B; \quad \frac{1}{N} \tilde{\Phi}_N' \tilde{\Phi}_N \rightarrow R + \Sigma,$$
The algorithm (15) is modified to

$$
\lim_{N \to \infty} \left( \theta_N - \theta \right) = (R + \Sigma)^{-1}(B - \Sigma\theta) \quad \text{w.p.1.}
$$

**Proof.** This follows from

$$
\theta_N - \theta \to (R + \Sigma)^{-1}(A + B) - R^{-1}A
$$

$$
= (R + \Sigma)^{-1}A - R^{-1}A + (R + \Sigma)^{-1}B
$$

$$
= -(R + \Sigma)^{-1}\Sigma R^{-1}A + (R + \Sigma)^{-1}B
$$

$$
= (R + \Sigma)^{-1}(B - \Sigma\theta).
$$

This completes the proof. \(\square\)

Identification bias can be corrected if \(\Sigma\) and \(B\) are known. The algorithm (15) is modified to

$$
\theta_N = \left( \frac{1}{N} \Phi_N^T \Phi_N - I \right)^{-1} \left( \frac{1}{N} \Phi_N^T Y_N - B \right).
$$

(16)

It follows from Theorem 1 that this modified \(\theta_N\) has a desired convergence property. If \(\Sigma\) and \(B\) are unknown, we can use statistical methods to estimate them. Then in the bias correction algorithm (16), in place of the true \(\Sigma\) and \(B\), we may use their estimates.

**Theorem 2:** Under the assumptions of Theorem 1, the estimates in (16) satisfy

$$
\theta_N \to \theta \quad \text{w.p.1 as } N \to \infty.
$$

**Proof.** By the strong law of large numbers, as \(N \to \infty\),

$$
\theta_N \to (R + \Sigma - \Sigma)^{-1}(A + B - B) = R^{-1}A = \theta \quad \text{w.p.1.}
$$

\(\square\)

### E. Recursive Algorithms for Bias Corrected LS Algorithms

We now introduce a recursive algorithm for (16).

**Theorem 3:** The estimates \(\theta_N\) in (16) can be updated recursively as

$$
\theta_N = \theta_{N-1} + K_N \left( I + \left( \frac{\phi' N}{\Sigma} \right) \theta_{N-1} \right)^{-1} P_{N-1}\phi_N - I
$$

$$
K_N = P_{N-1}\phi_N, \quad P_N = P_{N-1} - K_N \left( \frac{\phi' N}{\Sigma} \right) P_{N-1}
$$

**Proof:** From (16),

$$
\theta_N = \left( \frac{1}{N} \Phi_N^T \Phi_N - I \right)^{-1} \left( \frac{1}{N} \Phi_N^T Y_N - B \right)
$$

$$
= \left( \Phi_N^T \Phi_N - N\Sigma \right)^{-1} \left( \Phi_N^T Y_N - NB \right).
$$

Let \(P_N = (\Phi_N^T \Phi_N - N\Sigma)^{-1}\). Since \(\Phi_N^T \Phi_N = \Phi_N^T \Phi_N - 1 + \phi_N\phi_N'^T\), we have

$$
P_N = (\Phi_N^T \Phi_N - N\Sigma)^{-1}
$$

$$
= (\Phi_N^T \Phi_N - 1 + \phi_N\phi_N'^T - N\Sigma)^{-1}
$$

$$
= (P_{N-1} + \phi_N\phi_N'^T - N\Sigma)^{-1}
$$

$$
= \left( P_{N-1} + [\phi_N, -I] \left( \frac{\phi_N}{\Sigma} \right) \right)^{-1}
$$

By the matrix inversion lemma

$$
P_N = P_{N-1} - P_{N-1} [\phi_N, -I] \left( I + \left[ \frac{\phi' N}{\Sigma} \right] P_{N-1} [\phi_N, -I] \right)^{-1} \left[ \frac{\phi_N}{\Sigma} \right] P_{N-1}
$$

(17)

Moreover,

$$
\Phi_N^T Y_N - NB
$$

$$
= \Phi_N^T Y_N - (N-1)B + \phi_Ny_N - B
$$

$$
= \Phi_N^T Y_N - (N-1)B + [\phi_N, -I] \left( \frac{y_N}{B} \right)
$$

Define

$$
K_N = P_N [\phi_N, -I]
$$

By (17)

$$
K_N = P_N [\phi_N, -I]
$$

$$
= P_{N-1} [\phi_N, -I] \left( I + \left[ \frac{\phi' N}{\Sigma} \right] P_{N-1} [\phi_N, -I] \right)^{-1} \left[ \frac{\phi_N}{\Sigma} \right] P_{N-1}
$$

It follows that

$$
P_N = P_{N-1} - K_N \left( \frac{\phi_N}{\Sigma} \right) P_{N-1}
$$

Finally,

$$
\theta_N = P_N \left( \Phi_N^T Y_N - NB \right)
$$

$$
= (P_{N-1} - K_N \left( \frac{\phi_N}{\Sigma} \right) P_{N-1}) (\Phi_N^T Y_N - (N-1)B + [\phi_N, -I] \left( \frac{y_N}{B} \right))
$$

$$
= \theta_{N-1} - K_N \left( \frac{\phi_N}{\Sigma} \right) \theta_{N-1} + K_N \left( \frac{y_N}{B} \right)
$$

$$
= \theta_{N-1} + K_N \left( \frac{y_N}{B} \right) - \left[ \frac{\phi_N}{\Sigma} \right] \theta_{N-1}
$$

\(\square\)

### IV. Simulation Studies

Although we use a specific battery model structure in this paper, the main methodologies introduced in this paper are generic and can be applied to different battery models. In this section, we first illustrate the utility of bias correction algorithms by using some models of various structures and complexity. Then the specific circuit model is used for more detailed evaluations.

#### A. Illustrative Examples on Identification Bias Correction

We now illustrate effectiveness of the bias correction mechanisms described in the previous section.

**Example 1:** This example involves an FIR (finite impulse response) system of order 2 and one step delay: \(y_k = 0.5y_{k-1} - 2u_{k-2}\). The true parameter is \(\theta = [0.5, -2]'\). The input \(u_k\) and the corresponding output \(y_k\) are shown in the top plot of Figure 3. Suppose that the input is corrupted by an independent and identically distributed (i.i.d.) gaussian noise of mean zero and variance 0.01, and the output is subject to an i.i.d. gaussian measurement noise of mean zero
and variance 0.16. Input and output noises are independent. In this example, $B = 0$ and $\Sigma = 0.01I_2$ where $I_2$ is the $2 \times 2$ identity matrix. Without direct noise correction, identification errors demonstrate a persistent bias of norm 0.6373 (averaged Euclidean norm), with the exit estimate $\hat{\theta} = [-0.4041 - 1.1015]'$. With direct bias correction, such a bias is substantially reduced to an error norm 0.0156 and the exit estimate $\hat{\theta} = [0.5182, -2.0253]'$. These are shown in the bottom plot of Figure 3.

![Figure 3. Comparison of identification algorithms with and without bias correction for a second-order FIR system with a delay](image)

**Example 2:** This example involves an ARX (auto-regression with external input) system:

$$y_k - 0.4y_{k-1} - 0.3y_{k-2} = 1.493u_{k-1} - 0.298u_{k-2} + 0.1u_{k-3}.$$  

The true parameter is $\theta = [0.4, 0.3, 1.493, -0.298, 0.1]'$. The input $u_k$ and the corresponding output $y_k$ are shown in the top plot of Figure 4. The input is corrupted by an i.i.d. gaussian noise of mean zero and variance 0.04, and the output is subject to an i.i.d. gaussian measurement noise of mean zero and variance 0.01. Input and output noises are independent. In this example, $B = 0$ and

$$\Sigma = \begin{bmatrix} 0.01I_2 & 0 \\ 0 & 0.04I_3 \end{bmatrix}.$$  

Under a data window of size $N = 2000$, without direct noise correction, identification errors demonstrate a persistent bias of norm 0.1778, with the exit estimate $\hat{\theta} = [0.4272, 0.2802, 1.2581, 0.1249, -0.1219]'$.

With direct bias correction, such a bias is reduced to an error norm 0.0061 and the exit estimate $\hat{\theta} = [0.3924, 0.3074, 1.5051, -0.3008, 0.0917]'$.

These are shown in the bottom plot of Figure 4.

### B. Identification of Circuit Models for Battery Systems

The circuit model of the nominal parameter values (5) is simulated in a Simulink™ Model shown in Figure 5.

The Simulink™ model generated the current and voltage profiles shown in Figure 6, which will be used for parameter estimation. It should be emphasized here that to enhance identification quality, a very small random dither has been added to the load (current). This is indicated in the Simulink™ model by a random signal generator, and reflected in the signal profiles with noise-like signals. No measurement noises are added in the profiles at this point.

![Figure 4. Comparison of identification algorithms with and without bias correction for a third-order ARX system](image)

**Fig. 4. Comparison of identification algorithms with and without bias correction for a third-order ARX system**

**Fig. 5. Battery Model in Simulink™**

**Fig. 6. Current and voltage profiles of the battery**

The true system parameter in the modified regression structure (11) is

$$\theta = [a_1, a_2, b_1, b_2]' = [-1.9974, 0.9974, 6.6964, -6.6849]'$$

The estimated system by using the LS (or equivalently RLS) algorithms produced the parameter estimation for $\theta$

$$\hat{\theta} = [-1.9999, 0.9999, 6.6964, -6.6964]'$$

Due to added dithers, the estimates are highly accurate, even with a very small data window of size 50.

Although the estimation is of high fidelity in this case, estimation quality deteriorates substantially when measurement noises are introduced. When a very small measurement noise, a Gaussian noise of mean 0 and variance 0.00001, is added to the voltage measurements, parameter estimation loses its accuracy, shown in Figure 7. The identification algorithms are then modified to include bias correction. Figure
8 demonstrates substantially improved identification accuracy after bias correction.

![Parameter estimation errors increase when voltage measurements are slightly noise corrupted](image1.png)

![Parameter estimation becomes more accurate when bias correction algorithms are implemented](image2.png)

**V. CONCLUDING REMARKS**

This paper introduces identification methods for real-time battery model updates. The methods capture time-varying model parameters which can then be used for real-time control, optimization, diagnosis, and management. Due to some unique features in battery model structures, battery model identification encounters some challenging issues, such as identifiability, bias, and convergence. Modified identification algorithms are introduced to resolve these issues. Applications of the methods are demonstrated in some typical models in this paper. Much broader utilities are currently under investigation in different aspects of battery management systems. Also, discussions of this paper are limited to a single battery cell. Extensions to a battery pack consisting of a network of battery cells remain an open and critically important scenario for investigation.

**REFERENCES**


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