Cybernetics and Systems: An International Journal, 36: 513–526

ISSN: 0196-9722 print/1087-6553 online
DOI: 10.1080/01969720590944285

MULTIPLE SLIDING SURFACE CONTROL FOR SYSTEMS IN NONLINEAR BLOCK CONTROLLABLE FORM

ZHAO LU
FENG LIN
HAO YING

Department of Electrical and Computer Engineering, Wayne State University, Detroit, Michigan, USA

It is well-known that the multiple sliding surface control was developed to eliminate the problem of "explosion of complexity" inherent in the celebrated backstepping method. In this paper, we extend the multiple sliding surface control method further for systems in nonlinear block controllable form. As an application of the proposed control scheme, the robust attitude control law was synthesized for highly maneuverable missiles. Contrary to the classical SISO methods, all nonlinearities of missile attitude dynamics as well as coupling effects between roll, yaw and pitch channels due to roll rate are fully accommodated in the designed nonlinear control law.

INTRODUCTION

During the past several decades, a number of various mathematical techniques were propounded in the area of nonlinear robust control. In the early 1990s, a systematic approach, backstepping, was developed as a recursive procedure of designing stable control laws for a broad range of nonlinear systems (Krstic, Kanellakopoulos, and Kokotovic 1995). Backstepping is an approach that combines Lyapunov stability theory with the substantial advances made in nonlinear differential-geometric

Address correspondence to Z. Lu, Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202 USA. E-mail: zlu@eng.wayne.edu

control theory in the recent past. The basic concept behind backstepping is to use some states as fictitious controls to control other states. The primary potential benefit of this type of controller is that it allows a wide spectrum of nonlinearities to be incorporated in the controller design, and has proven nominal stability and convergence of error.

However, the computational complexity in backstepping design procedure naturally increases with the system order, that is, it suffers from an "explosion of complexity" due to the necessity to perform repeated differentiations of nonlinear functions. The attempt to overcome such a drawback, as well as strengthen the robustness of the designed control, motivated the combination of the basic backstepping procedure with other robust control techniques, particularly, the sliding mode control, which is distinguished for its invariance against matched uncertainties of various kinds, such as parameter variations and external disturbances when the controlled system is in sliding mode.

To this end, a procedure similar to backstepping, called Multiple Sliding Surface (MSS) control was proposed to simplify the controller design of systems where model differentiation was difficult (Hedrick and Yip 2000; Won and Hedrick 1996). Multiple sliding surface control has similar interlaced structures and stipulates the strict feedback assumption of Single-input Single-output (SISO) nonlinear system. This approach has been implemented successfully and has shown an advantage over the conventional sliding mode control.

In this article, the MSS control was developed further for a class of Multi-input Multi-output (MIMO) nonlinear systems, which could be formulated or transformed into Nonlinear Block Controllable (NBC) form (Luk'yanov 1998). The regular form of affine control system is composed of the controlled and uncontrolled subsystems, where the state space vector of the controlled subsystem is considered as fictitious input control vector for the uncontrolled subsystem, and in addition, the unmatched uncertainty for the full system is matched for the uncontrolled subsystem with respect to the fictitious control input. The nonlinear block controllable form consists of a set of blocks with a structure similar to that of the controlled dynamics of the regular form, that is, the block state vector and the block fictitious control vector have the same dimension. Thus, the NBC form can be interpreted as a generalization of the regular form. Such a kind of representation enables to reduce the original control law synthesis problem into a sequence of low-order subproblems.

In the previous efforts to stabilize the systems in nonlinear block controllable form, a nonlinear transformation of the state variables was required to linearize the nominal part of the systems, by which a switching function was designed in the transformed coordinates (Loukianov 2002a; Loukianov, Toledo, and Dodds 2002b). In contrast to that, the linearization and nonlinear transformation are removed in our method, and the robust tracking control law is designed in a very straightforward and effective way. Particularly, high robustness to parametric uncertainties is guaranteed.

As an application of the proposed control scheme, the attitude control problem of Bank-to-Turn (BTT) missiles with parametric uncertainties was considered. The BTT missiles have higher maneuverability over Skid-to-Turn (STT) missiles and hence the autopilot design for BTT missiles has been widely studied. Since the direction of the aerodynamic normal force is determined by rolling the missile, the BTT missile offers the possibility of improved storage and aerodynamic performance. Moreover, the BTT missile may be designed to use an airbreathing propulsion system providing a greater range for a given weight of fuel. Proper operation of an airbreathing missile, however, constrains the side-slip angle to be very small (several degrees, at most). The reason for this is that a large value can lead to that the engine does not get enough air and goes out. Therefore, the BTT missile means that it turns by first rolling followed by the acceleration in the negative pitch channel due to the placement of the air inlet. By performing a turn in this way, the side-slip angle is kept small so that the engine can be provided with air. So, if the missile is flying straight forward and a strong dive is desired, the missile must first roll over on its back, followed by the acceleration in the negative pitch channel (Lee, Lee, and Ha 2001).

For the BTT missile, the associated aerodynamics are highly nonlinear and a high roll rate combined with the asymmetric structure of the missile body produces severe undesirable coupling between roll, yaw, and pitch channels. Therefore, the autopilot design of a modern BTT is a challenging task.

In the past, the classical SISO methods, which neglect the nonlinear terms to get independent pitch and yaw channels or which obtain linear approximation of the cross-coupling at each design point, have been extensively used for the BTT autopilot design. These methods are only adequate for small roll rates. Even for the linear autopilot design with gain scheduling, the design process inevitably suffers from time consuming and tedious work. Contrary to decoupling the attitude dynamics into independent

SISO channels in the conventional methods, the coupling effect between roll, yaw, and pitch channels in BTT missiles was accommodated and fully considered in synthesizing the multiple sliding surface control law in our work. Numerical simulation on attitude control of BTT missiles demonstrates the effectiveness of the synthesized nonlinear robust control law.

MULTIPLE SLIDING SURFACE CONTROL FOR SYSTEMS IN NBC FORM

Let us consider a class of nonlinear uncertain plants described by

$$\dot{x} = f(x) + B(x)u(t) + \Delta f(x), \tag{1}$$

where x is an n-dimensional state vector, and u is an m-dimensional control vector; the elements of the vector f and of the matrix B are smooth functions of all their arguments. By using the integral transformation method, system (1) can be reduced to the following nonlinear block controllable form (Luk'yanov 1998).

$$\begin{cases} \dot{x}_1 = f_1(x_1) + B_1(x_1)x_2 + \Delta f_1(x_1) \\ \dot{x}_i = f_i(x_1, x_2, \ldots, x_i) + B_i(x_1, x_2, \ldots, x_i)x_{i+1} + \Delta f_i(x_1, x_2, \ldots, x_i), \\ \qquad\qquad i = 2, \ldots, r-1 \\ \dot{x}_r = f_r(x_1, x_2, \ldots, x_r) + B_r(x_1, x_2, \ldots, x_r)u + \Delta f_r(x_1, x_2, \ldots, x_r), \end{cases} \tag{2}$$

where $x = \mathrm{col}(x_1, \ldots, x_r)$, $x_i \in X_i \subseteq R^{n_i}$, and $i = 1, \ldots, r$. Note that the state equation for $\dot{x}_i$ depends both on $x_1, \ldots, x_i$ and affinely on x_{i+1}. Conditions for converting general nonlinear systems into the NBC form (2) are given in recent literature (Luk'yanov 1998).

Definition 1: System (2) is called a nonlinear block controllable form for system (1) if, in every i-th block of (2), the matrix $B_i(x_1, x_2, \ldots, x_i)$ positioned before the vector x_{i-1}, which serves as a fictitious control vector in this block is of full row rank, that is,

$$\dim X_i = \mathrm{rank}\ B_i(x_1, \ldots, x_i) = n_i, \tag{3}$$

where the integers $(n_1, n_2, \ldots, n_r)$ are the controllability indices of system (1) with

$$\sum_{i=1}^{r} n_i = n, \tag{4}$$

and $n_1 \leq n_2 \leq \ldots \leq n_r \leq m$.

It is worth noting that the dynamics of many plants (for instance, electromechanical systems) can be written in the NBC form (2). Furthermore, the output of the plant coincides with the state vector of the first block. In this article, it is assumed that all state variables are available for measurement.

The objective of the controller is to make x_1 track a desired trajectory $x_{1d}(t)$. To extend the MSS control method to system (2) in NBC form, the first sliding surface is defined as:

$$S_1 = x_1 - x_{1d}. \tag{5}$$

The dynamics of S_1 can be described by

$$\dot{S}_1 = f_1(x_1) + B_1(x_1)x_2 + \Delta f_1(x_1) - \dot{x}_{1d}. \tag{6}$$

If x_2 is considered as the forcing term for the surface dynamics, then $S_1^T \dot{S}_1 < 0$ is satisfied if $x_2 = x_{2d}$, where

$$x_{2d} = B_1^+(x_1)[\dot{x}_{1d} - f_1(x_1) - K_1 S_1]. \tag{7}$$

Here $B_1^+(x_1)$ stands for the right pseudo-inverse of $B_1(x_1)$, that is,

$$B_1^+(x_1) = B_1^T(x_1)\left[B_1(x_1)B_1^T(x_1)\right]^{-1}. \tag{8}$$

Now the next step is to make x_2 track x_{2d}, so define the second sliding surface as $S_2 = x_2 - x_{2d}$, and then x_{3d} is chosen to drive S_2 to zero so that x_2 can track x_{2d}.

Proceeding similarly, define i-th surface as

$$S_i = x_i - x_{id}. \tag{9}$$

The corresponding dynamics of the i-th surface can be described by

$$\dot{S}_i = f_i(x_1, x_2, \ldots, x_i) + B_i(x_1, x_2, \ldots, x_i)x_{i+1} + \Delta f_i(x_1, x_2, \ldots, x_i) - \dot{x}_{id}. \tag{10}$$

$x_{i+1,d}$ was chosen to drive S_i to zero:

$$x_{i+1,d} = B_i^+(x_1, x_2, \ldots, x_i)[\dot{x}_{id} - f_i(x_1, x_2, \ldots, x_i) - K_i S_i], \tag{11}$$

where $B_i^+(x_1, x_2, \ldots, x_i)$ denotes the right pseudo-inverse of $B_i(x_1, x_2, \ldots, x_i)$, that is

$$B_i^+(x_1, x_2, \ldots, x_i) = B_i^T(x_1, x_2, \ldots, x_i)[B_i(x_1, x_2, \ldots, x_i)B_i^T(x_1, x_2, \ldots, x_i)]^{-1}. \tag{12}$$

After continuing this procedure up to $i = n - 1$, define $S_n = x_n - x_{nd}$. Finally, the desired control input is synthesized as

$$u = B_r^+(x_1, x_2, \ldots, x_r)[\dot{x}_{rd} - f_r(x_1, x_2, \ldots, x_r) - K_r S_r]. \tag{13}$$

Obviously, the difficulty with this scheme lies in finding the time derivatives of reference trajectories for the i-th state, i.e., the synthetic input $\dot{x}_{id}, i = 1, 2, \ldots, r$. This problem has been dealt with in an ad hoc way by numerical differentiation (Hedrick and Yip 2000):

$$\dot{x}_{id}(n) \approx \frac{x_{id}(n) - x_{id}(n-1)}{\Delta T}, \tag{14}$$

where ΔT is the sample period. In Swaroop, Hedrick, Yip, and Gerdes, (2000), the use of a low-pass filter was discussed to smooth the signal produced by Eq. (14). This ad hoc approach has worked well in many experimental applications ranging from active suspension control (Alleyne and Hedrick 1995) to fuel-injection control (Choi, Won, and Hedrick 1994), and so on. Therefore, the synthesized control law can be written as

$$u = B_r^+(x_1, x_2, \ldots, x_r)\left[\frac{x_{rd}(n) - x_{rd}(n-1)}{\Delta T} - f_r(x_1, x_2, \ldots, x_r) - K_r S_r\right]. \tag{15}$$

This control law design procedure does not involve model differentiation and linearization and thus has prevented the explosion of the terms and avoided the use of nonlinear coordinate transformation.

ROBUST ATTITUDE CONTROL OF THE MISSILE

The utility of the proposed control strategy is demonstrated through attitude control of BTT missiles in this section. The nonlinear attitude dynamics of generic, highly maneuverable BTT missiles can be expressed in state-space form as follows (Fu, Chang, Yang, and Kuo 1997; Lian, Fu, Chuang, and Kuo 1994):

$$\begin{cases} \dot{\phi} = \omega_x + (\omega_y \sin\phi + \omega_z \cos\phi)\tan\theta \\ \dot{\theta} = \omega_y \cos\phi - \omega_z \sin\phi \\ \dot{\psi} = (\omega_y \sin\phi + \omega_z \cos\phi)/\cos\theta \\ \dot{\omega}_x = \dfrac{J_y - J_z}{J_x}\omega_y\omega_z + \dfrac{1}{J_x}M_x \\ \dot{\omega}_y = \dfrac{J_z - J_x}{J_y}\omega_z\omega_x + \dfrac{1}{J_y}M_y \\ \dot{\omega}_z = \dfrac{J_x - J_y}{J_z}\omega_x\omega_y + \dfrac{1}{J_z}M_z, \end{cases} \tag{16}$$

where ω_x, ω_y and ω_z are the roll-angular rate, pitch-angular rate, and yaw-angular rate in the body-fixed frame, and ϕ, θ, and ψ are roll angle, pitch angle, and yaw angle with respect to the inertial frame; (J_x, J_y, J_z) and (M_x, M_y, M_z) are the moments of inertia and rolling, pitching, and yawing torques, respectively, are all about the body frame.

The equations describing the attitude control problem are basically those of a rotating rigid body with extra terms describing the effect of the control torques. They therefore consist of kinematic equations relating the angular position with the angular velocity, and dynamic equations describing the evolution of angular velocity or, equivalently, angular momentum.

By defining $x_1 = (\phi, \theta, \psi)^T$, $x_2 = (\omega_x, \omega_y, \omega_z)^T$, and $p = (J_x, J_y, J_z)^T$, this attitude dynamics of missile (15) could be written into the following NBC form, consisting of two blocks:

$$\begin{cases} \dot{x}_1 = G_1(x_1)x_2 \\ \dot{x}_2 = f(x_2, p) + G_2(p)u, \end{cases} \tag{17}$$

where

$$G_1(x_1) = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{pmatrix}, \tag{18}$$

$$G_2(p) = \begin{pmatrix} \frac{1}{J_x} & & \\ & \frac{1}{J_y} & \\ & & \frac{1}{J_z} \end{pmatrix}, \qquad f(x_2, p) = \begin{pmatrix} \dfrac{J_y - J_z}{J_x}\omega_y\omega_z \\ \dfrac{J_z - J_x}{J_y}\omega_z\omega_x \\ \dfrac{J_x - J_y}{J_z}\omega_x\omega_y \end{pmatrix},$$

$$u = \left[M_x, M_y, M_z\right]^T.$$

Apparently, the attitude dynamics depends on the inertial moments parameter p, which can vary over a wide range, depending on the mass variations of the missile due to fuel usage, i.e., $p = p_0 + \Delta p$. It is necessary for the control system to be able to adapt to these variations and uncertainties, which poses a formidable challenge to the controller design. It is worth noting that $G_2(p)$ keeps being a positive definite diagonal matrix even though the inertial moments parameter p varies, which will be utilized to simplify the controller design.

It can be inferred that $f(x_2, p)$ can be decomposed as

$$f(x_2, p) = \begin{pmatrix} \frac{1}{J_x} & & \\ & \frac{1}{J_y} & \\ & & \frac{1}{J_z} \end{pmatrix} \begin{pmatrix} (J_y - J_z)\omega_y\omega_z \\ (J_z - J_x)\omega_z\omega_x \\ (J_x - J_y)\omega_x\omega_y \end{pmatrix} = G_2(p)\tilde{f}(x_2, p). \quad (19)$$

Therefore, the second block subsystem in system (17) can be rewritten as

$$\dot{x}_2 = G_2(p)\tilde{f}(x_2,\ p) + G_2(p)u, \quad (20)$$

which manifests that $f(x_2,\ p)$ can be viewed as the matched uncertainty in the second block subsystem of the system (17).

First, we consider the first block subsystem in the system (17):

$$\dot{x}_1 = G_1(x_1)x_2. \quad (21)$$

To make the angular state $x_1 = (\phi, \theta, \psi)^T$ follow the commanded attitude reference $x_{1d} = (\phi_d, \theta_d, \psi_d)^T$, the first sliding surface is defined as

$$S_1 = x_1 - x_{1d}. \quad (22)$$

Hence, the dynamics of S_1 is

$$\dot{S}_1 = G_1(x_1)x_2 - \dot{x}_{1d}. \quad (23)$$

To make the reaching condition $S_1^T\dot{S}_1 < 0$ hold, a reasonable choice for x_{2d} is

$$x_{2d} = G_1^{-1}(x_1)[-K_1S_1 + \dot{x}_{1d}], \quad (24)$$

where

$$G_1^{-1}(x_1) = \begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix}. \quad (25)$$

Next the goal here is to find a robust control law $u(t)$ to guide x_2 to match the prescribed reference signal x_{2d} against the existing parametric uncertainties Δp. Define the second sliding surface as

$$S_2 = x_2 - x_{2d}, \tag{26}$$

and its dynamics can be expressed by

$$\dot{S}_2 = G_2(p)u + G_2(p)\tilde{f}(x_2, p) - \dot{x}_{2d}. \tag{27}$$

The control input $u(t)$ need to be designed to regulate S_2 to zero. On condition that the input matrix $G_2(p)$ is a diagonal matrix, the term $G_2(p)\tilde{f}(x_2, p) - \dot{x}_{2d}$ can be viewed as matched uncertainties in this surface dynamics, i.e., the nominal surface dynamics of (27) is

$$\dot{S}_2 = G_2(p)u. \tag{28}$$

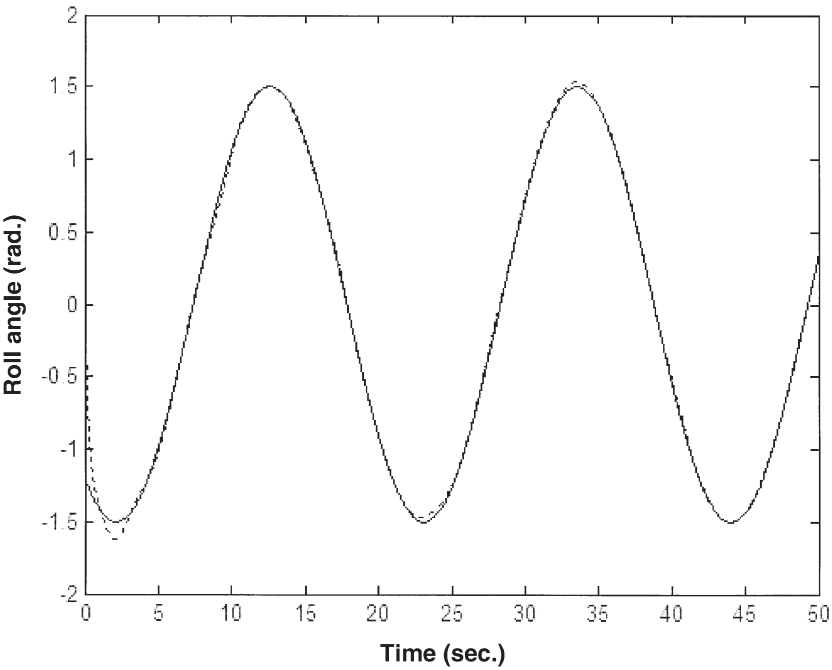

Figure 1. Tracking performance of roll angle $\phi(t)$ to the commanded roll angle $\phi_d(t)$ (solid line: $\phi_d(t)$, dotted line: $\phi(t)$).

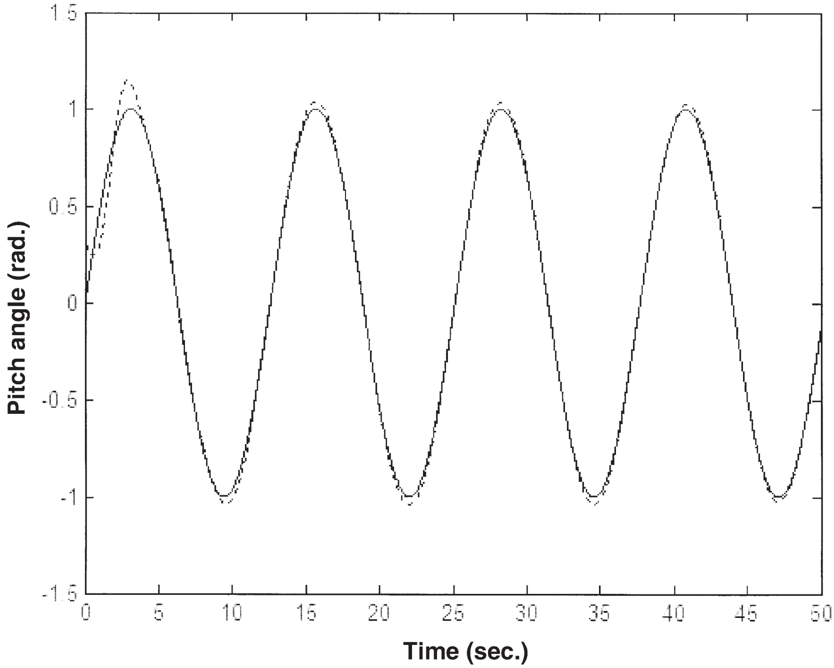

Figure 2. Tracking performance of pitch angle $\theta(t)$ to the commanded pitch angle $\theta_d(t)$ (solid line: $\theta_d(t)$, dotted line: $\theta(t)$).

Therefore, to stabilize the nominal surface dynamics (28), u can be designed as

$$u = -K_2 S_2, \tag{29}$$

where K_2 is a positive definite diagonal matrix, which can be chosen as a high-gain to make the system converge fast towards the desired manifold S_2. The complete attitude control law is obtained by substituting (22), (24), and (26) into (29).

$$u = -K_2 \{ x_2 - G_1^{-1}(x_1)[-K_1(x_1 - x_{1d}) + \dot{x}_{1d}] \}. \tag{30}$$

The asymptotical stability of the nominal surface dynamics of (28) could be shown by substituting (29) into the nominal surface dynamics (28) due to the fact that $G_2(p)$ and K_2 are both positive definite diagonal matrix.

Usually a control system is designed under the assumption that a missile has entered a burn-out state, and also missile mass and inertial

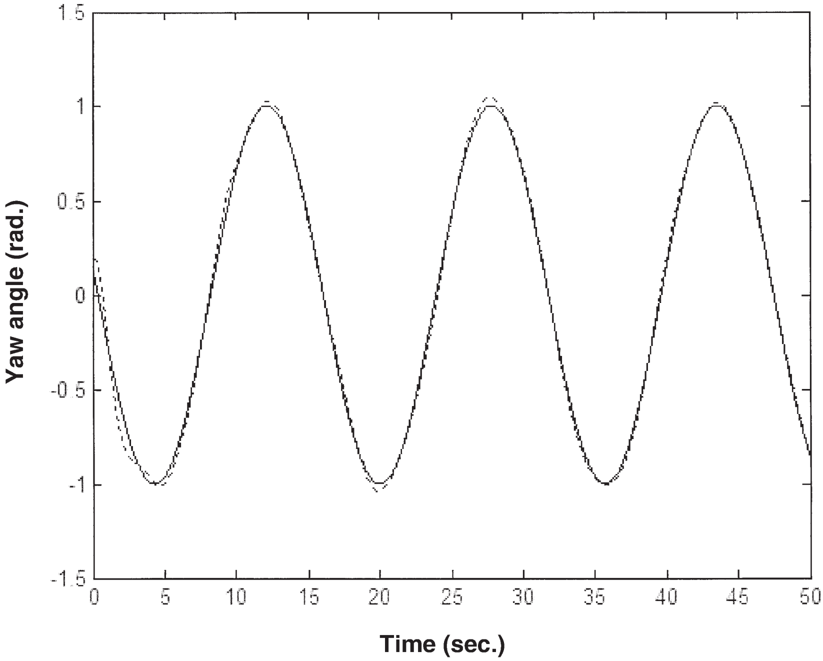

Figure 3. Tracking performance of yaw angle $\psi(t)$ to the commanded yaw angle $\psi_d(t)$ (solid line: $\psi_d(t)$, dotted line: $\psi(t)$).

moments are constant. However, in our simulation, for the purpose of verifying the robustness of the control strategy, the variations on inertial moments are included. The inertial moments matrix is assumed to be decreased linearly to half of their initial values until the missile hits the target (Choi, Chwa, and Kim 2000), i.e.,

$$J(t) = J(t_0) - \frac{J(t_0)}{2} \cdot \frac{t}{t_s}, \tag{31}$$

where t_0 is the launching time and t_s is the ending time. The inertial moments matrix $J(t)$ is defined as

$$J(t) = \mathrm{diag}([J_x(t), J_y(t), J_z(t)]^T), \tag{32}$$

and the initial inertia matrix $J(t_0)$ is chosen as

$$J(t_o) = \mathrm{diag}([J_x(t_0), J_y(t_0), J_z(t_0)]^T) = \mathrm{diag}([0.5, 80, 100]^T)\mathrm{kg\ m}^2, \tag{33}$$

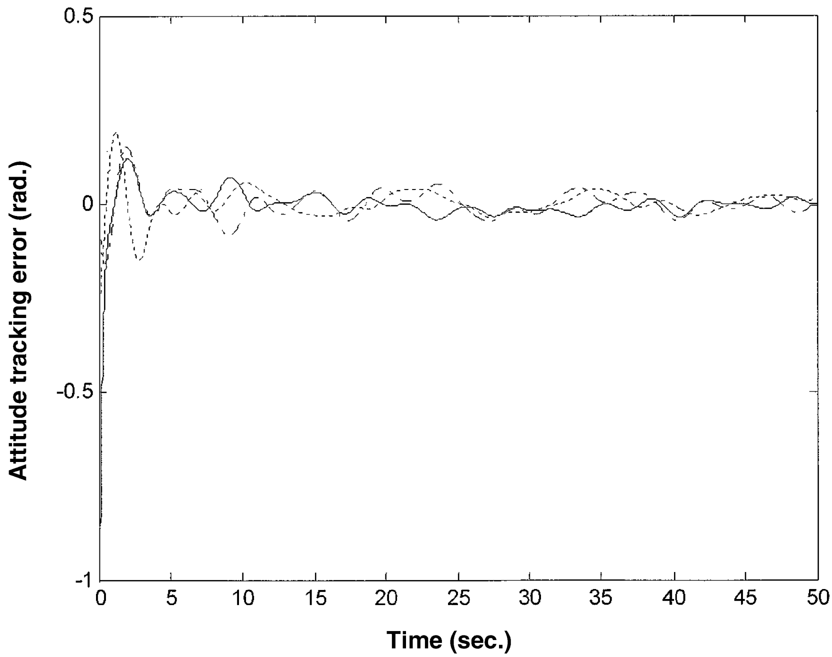

Figure 4. Tracking errors versus time (solid line: $\phi_d - \phi$, dotted line: $\theta_d - \theta$, dash-dot line: $\psi_d - \psi$).

which characterizes the asymmetric structure of missile body. The initial attitude angles and angular velocities are

$$\begin{aligned} [\phi(t_0), \theta(t_0), \psi(t_0)]^T &= [0.2, 0.3, 0.2]^T \text{rad} \\ \omega(t_0) = [\omega_x(t_0), \omega_y(t_0), \omega_z(t_0)]^T &= [0, 0, 0]^T \text{rad}/s. \end{aligned} \tag{34}$$

The reference for attitude commands is specified as $\phi_d(t) = 1.5\cos(0.3t + 2.5)$, $\theta_d(t) = \sin(0.5t)$, and $\psi_d(t) = \sin(0.4t + 3)$. The controller parameters are chosen as $K_1 = \text{diag}([5, 5, 5]^T)$ and $K_2 = \text{diag}([50, 100, 100]^T)$. The tracking control performance under the control law defined by (30) and (25) is visualized by Figures 1–3 and the time response of tracking errors is illustrated in Figure 4, which show that the angular states follow the attitude commands with fast transient and satisfactory performance in spite of inertia uncertainty, while Figure 5 demonstrates the control input signals.

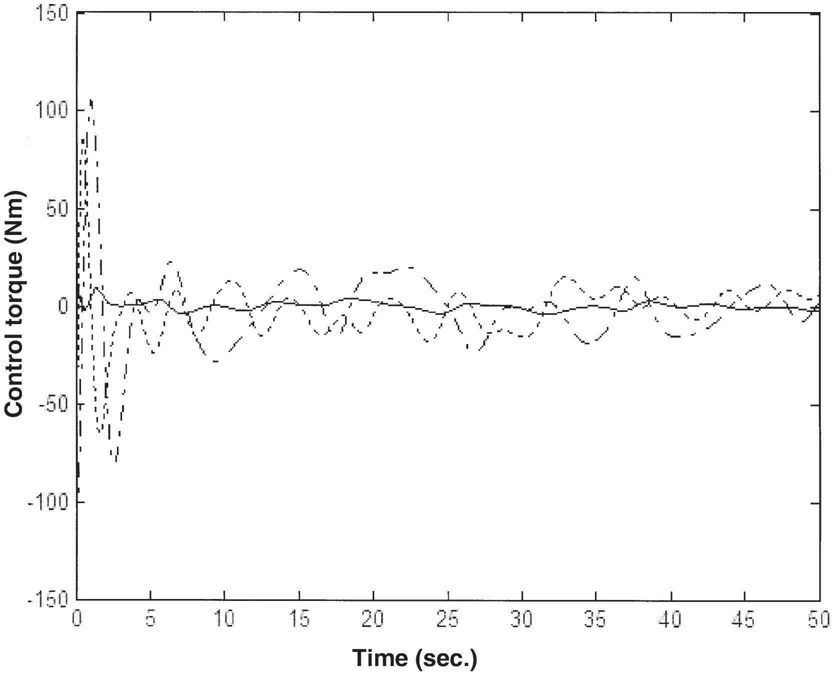

Figure 5. Control input $u(t)$ versus time (solid line: $M_x(t)$, dotted line: $M_y(t)$, dash-dot line: $M_z(t)$).

CONCLUSIONS

A criticism for the well-known backstepping methods is that there is an explosion of the number of terms that follow as the iteration steps increase and there is a need to differentiate the initial functions many times. An alternative that has been proposed called MSS control is able to eliminate the problem of "explosion of complexity" inherent in the backstepping method. Our development in this article is to broaden the application field of the multiple sliding surface control to a class of MIMO nonlinear systems, that is, the systems in nonlinear block controllable form. As an example, we applied the proposed control scheme in attitude control of BTT missile with parametric uncertainties. Particularly, the derived control law was simplified greatly by utilizing the special structure in missile attitude dynamics, which demonstrates the flexibility of the proposed control method. In spite of its ease of implementation, the tracking performance with high accuracy was achieved.

REFERENCES

Alleyne, A. and J. K. Hedrick. 1995. Nonlinear adaptive control of active suspensions. *IEEE Transactions on Control Systems Technology*, 3(1):94–101.

Choi, J. Y., D. Chwa, and M. S. Kim. 2000. Adaptive control for feedback-linearized missiles with uncertainties. *IEEE Transactions on Aerospace and Electronic Systems*, 36(2):467–481.

Choi, S. B., M. C. Won, and J. K. Hedrick. 1994. Fuel-injection control of S.I. engines. In *Proceedings of IEEE Control and Decision Conference*, pp. 1609–1614, 14–16 December, Florida, USA.

Fu, L. C. W. D. Chang, J. H. Yang, and T. S. Kuo. 1997. Adaptive robust Bank-to-Turn missile autopilot design using neural networks. *Journal of Guidance, Control, and Dynamics*, 20(2):346–354.

Hedrick, J. K. and P. P. Yip. 2000. Multiple sliding surface control: Theory and application. *Journal of Dynamic Systems, Measurement, and Control*, 122(4): 586–593.

Krstic, M., I. Kanellakopoulos, and P. Kokotovic. 1995. *Nonlinear and adaptive control design*. New York: John Wiley & Sons, Inc.

Lee, S. Y., J. I. Lee, and I. J. Ha. 2001. Nonlinear autopilot for high maneuverability of bank-to-turn missiles. *IEEE Transactions on Aerospace and Electronic Systems*, 37(4):1236–1253.

Lian, K. Y., L. C. Fu, D. M. Chuang, and T. S. Kuo. 1994. Nonlinear autopilot and guidance for a highly maneuverable missile. In *Proceedings of the American Control Conference*, pp. 2293–2297, 29 June–1 July, Maryland, USA.

Loukianov, A. G. 2002a. Robust block decomposition sliding mode control design. *Mathematical Problems in Engineering*, 8(4–5):349–365.

Loukianov, A. G., B. C. Toledo, and S. Dodds. 2002b. Robust stabilization of a class of uncertain system via block decomposition and VSC. *International Journal of Robust and Nonlinear Control*, 12(14):1317–1338.

Luk'yanov, A. G. 1998. Nonlinear block control with sliding mode. *Automation and Remote Control*, 59(7):916–933.

Swaroop, D., J. K. Hedrick, P. P. Yip, and J. C. Gerdes. 2000. Dynamic surface control for a class of nonlinear systems. *IEEE Transactions on Automatic Control*, 45(10):1893–1899.

Won, M. and J. K. Hedrick. 1996. Multi-surface sliding control of a class of uncertain nonlinear system. *International Journal of Control*, 64(4):693–706.