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Predictive fuzzy PID control: theory, design and simulation

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Abstract

Predictive fuzzy PID control theory is developed in this paper, which offers a new approach for robust control of time-delay systems. The paper describes the functional structure, design principle, and stability analysis of a new predictive fuzzy PID controller. Sufficient computer simulations are provided for illustration and verification. First, the structure of the controller is derived from both the fuzzy PID control and the generalized predictive control concepts. Then, on-line model identification, optimal cost index, fuzzification, rule base, and defuzzification of the representative predictive fuzzy PD+I controller are discussed in detail. Lyapunov asymptotic stability analysis is conducted. Then, many computer simulations are performed to compare with several closely related controllers such as the fuzzy PD+I controller and the Smith-type predictive fuzzy PD+I controller. In the simulations, second-order linear systems with/without time delays, nonlinear systems with/without time delays, uncertain linear systems with time delays, and uncertain nonlinear systems with time delays are used to confirm the advantages of the new predictive fuzzy PD+I controller. Finally, this method is applied to control some chaotic systems with success. This predictive fuzzy control method provides a new way for controlling uncertainty and complex linear and nonlinear systems, even with significant time delay. © 2001 Elsevier Science Inc. All rights reserved.

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1. Introduction

Predictive control is a control strategy that is based on the prediction of the plant output over an extended horizon in the future [1], which enables the controller to predict future changes of the measurement signal and to base control actions on the prediction [2]. There are many advantages of predictive control [1]. Due to these advantages, the predictive control method has many applications. Examples include medical applications [3], civil engineering applications [1], mechanical engineering applications [4], and chemical and petrochemical engineering applications [5].

Over the last decade, a lot of research has been carried out on predictive control. At present, predictive control research seems to focus on model-based predictive control, PID types of predictive control, fuzzy logic based predictive control, and neural-net predictive control.

Model-based predictive control [3] has become popular over the past 20 years as a powerful tool in feedback control for solving many problems for which other control approaches were proved to be ineffective. Interest in predictive control began in 1978 when Richalet [6] formulated an identification command controller. In 1979, Cutler and Ramaker [7] proposed a predictive control strategy, known as dynamic matrix control, which promised to deal with processes having large time delays. But self-tuning and adaptive control researchers were disappointed by the instability of these early control algorithms. In 1987, Clarke [26] proposed an generalized predictive control (GPC) which shows good robustness of adaptive model-based predictive control (MBPC). Then, GPC becomes the most popular one for several reasons: (i) the controlled autoregressive and integrated moving average model, on which the approach is based, is effective in counteracting the offset problem; (ii) the inclusion of control objective which takes into account future weighted errors together with weighted control effort; (iii) the flexibility for fine-tuning tailored to a particular application; (iv) the ability to include simultaneous filtering in control and estimation. But the finite horizon strategies of MBPC appeared to be difficult to analyze and general stability results were scarce. After that, GPC had been developed to stable generalized predictive control [8], multivariable stable generalized predictive control [9,10], constrained cautious stable predictive control [11,25], and constrained multivariable cautious stable predictive control [12]. Other predictive control methods have also been developed, which combine neural-net predictive model with GPC [13,14] or fuzzy predictive model with GPC [5]. All of these are model-based predictive control methods. The two main drawbacks of GPC that prevent it from widespread practical utilization are computational complexity and the need of a precise model.

On the other hand, PID controllers are widely used in industry. In order to meet the increasing demands on quality and economic performance of control systems, predictive PID control gains attention gradually. It has become a promising branch of predictive control research today [15–17].

Fuzzy control, as another alternative, is also widely used for controlling nonlinear systems whose plant models are unknown or vague. Combining it with predictive control may avoid model-based GPC's drawbacks. For this reason, fuzzy predictive control receives more and more attention recently [5,18,19].

As another trend, fuzzy PID control method has been developed during the last decade. It has analytic structure and guaranteed stability, and shows good performance for controlling general nonlinear systems [20–23,27,28].

Based on the above literature review, we thought that if we could combine fuzzy PID control with GPC to form a new predictive fuzzy PID control method, we could build the predictive and optimal features into the fuzzy PID controller. These features are expected to make the new predictive fuzzy PID controller more effective in handling nonlinear, complex, even uncertain and time-delay systems. This has been the main motivation for the present research.

2. Design of the predictive fuzzy PD+I controller

2.1. Basic principles of the predictive fuzzy PD+I controller

The proposed predictive fuzzy PID controller is shown in Fig. 1. It is composed of a CARMA model, a fuzzy PD controller, a fuzzy I controller, and an optimal cost index to be minimized on-line. The fuzzy PD and fuzzy I controllers form a fuzzy PID controller. Comparing with a direct formation of a fuzzy PID controller, our configuration has the advantage of being easier to analyze analytically.

In Fig. 1, $R_J(k)$ is the reference for the optimal cost index J, $y(k)$ is plant output, $u(k)$ is control input of the plant, $\hat{y}(k+1)$ is predictive value of y based on the predictive CARMA model, $R_y(k+1), R_y(k), \ldots, R_y(k-m)$ are the references of y in different time steps, and $\Delta u(k)$ is change of the controller output.

To achieve the objective of predictive control, we use the CARMA model as a predictor to predict $\hat{y}(k+1)$. To introduce some features that are similar to the features of GPC, we integrate the optimal cost index J into the controller directly. It may significantly reduce the computational complexity of GPC. In this structure, in fact, we can choose any optimal constraints in J and any non-predictive fuzzy controller. In this paper, we choose fuzzy PD+I controller only as a example.

To achieve stable predictive control, particularly for nonlinear time-delay systems, we reformulate the control problem to a tracking problem. There are

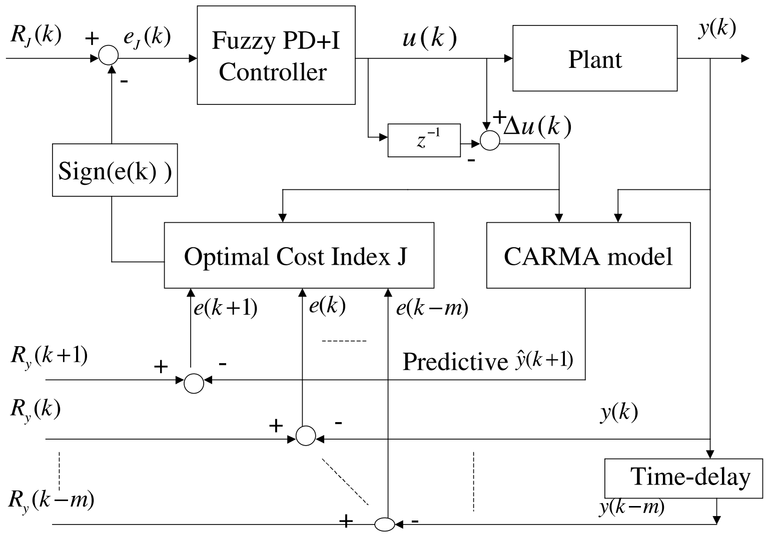

Fig. 1. The proposed predictive fuzzy PD+I control system.

two objectives in tracking. One is to drive the plant output to track a given plant-output reference. Another is to force the J to track a given J-reference, or make it as small as possible. To make sure the control system is stable in the sense of Lyapunov, we can choose the J-reference to be a monotonic decline curve or even a zero value. Thus, it will guide the change of J to be negative at most time steps. In the proposed predictive fuzzy PID controller, we modify the general optimal cost index J to include the tracking errors, rates of errors, and control changes. We use J-error and J-error-rate as inputs to the fuzzy PID controller. We then adjust the parameters of the fuzzy PID controller to force J to approximate the reference as time approaches infinity. On the other hand, we also need to drive the plant output y to approximate its reference, R_y, when we adjust the fuzzy PID controller so as to ensure J to approximate its reference, R_J. In other words, when the errors and error-rates of y both approach zero, J-error and J-error-rate must also approach zero. Therefore, the advantage of this scheme is that we only need to adjust the parameters of one fuzzy PID controller to achieve two objectives. It makes the whole controller predictive, optimal, robust and stable, even for uncertain nonlinear time-delay systems.

2.2. Identification of the ARMAX predictive model

Controlled auto-regressive moving average model (CARMA model) is used to represent the unknown or uncertain system, so that a stochastic framework can be taken into account and uncertain models of variable complexity can be considered. We will use an identification algorithm to adapt the system on-line. The formula of the CARMA model used here is

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}) y(k) \\ = (b_1 + b_2 z^{-1} + \cdots + b_m z^{-m+1}) u(k) + (c_0 + c_1 z^{-1} + \cdots + c_p z^{-p}) \xi(k) + d. \tag{1}$$

In this equation, a_i, b_i, and c_i are coefficient parameters, d is offset, $\xi(k)$ is noise, $u(k)$ is controller output, $y(k)$ is output of the plant, and m, n, and p are positive integers. If we suppose that the offset d is zero and noise $\xi(k)$ is zero, then (1) becomes

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}) y(k) = (b_1 + b_2 z^{-1} + \cdots + b_m z^{-m+1}) u(k). \tag{2}$$

Rearranging it in a matrix form, it becomes

$$\begin{bmatrix} -y(k-1) & -y(k-2) & \cdots & -y(k-n) & u(k) & u(k-1) & \cdots & u(k-m+1) \\ -y(k-2) & -y(k-3) & \cdots & -y(k-n-1) & u(k-1) & u(k-2) & \cdots & u(k-m) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ -y(k-p) & -y(k-p-1) & \cdots & -y(k-p-n+1) & u(k-p+1) & u(k-p) & \cdots & u(k-p-m+2) \end{bmatrix}_{p\times(m+n)}$$

$$\times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{(m+n)\times 1} = \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-n+1) \\ \vdots \\ \vdots \\ \vdots \\ y(k-p+1) \end{bmatrix}_{p\times 1}. \tag{3}$$

For notational convenience, we rewrite Eq. (3) as

$$A_{p\times(m+n)} X_{(n+m)\times 1} = Y_{p\times 1}, \tag{4}$$

where p is the total number of available y values, n is the total number of coefficients a_i need to be identified, and m is the total number of coefficients b_i need to be identified. We have three cases to consider:

1. When $p < (m+n)$, $\quad X_{(n+m)\times 1} = A'_{(m+n)\times p} \; * \; Y_{p\times 1}$.
2. When $p = (m+n)$, $\quad X_{(n+m)\times 1} = (A)^{-1}_{p\times(m+n)} \; * \; Y_{p\times 1}$.
3. When $p > (m+n)$, $\quad X_{(n+m)\times 1} = (A'A)^{-1}_{(m+n)\times(m+n)} \; * \; A'_{(m+n)\times p} \; * \; Y_{p\times 1}$.

2.3. Optimal cost index for stable predictive control

The optimal cost index is defined by

$$J = \sum_{i=-1}^{H}(r(k-i)-y(k-i))^2 + \sum_{j=0}^{H_c}(\Delta u(k-j))^2$$
$$= \sum_{i=-1}^{H} e^2(k-i) + \sum_{j=0}^{H_c}(\Delta u(k-j))^2, \qquad (5)$$

where r is the reference, $\hat{y}(k+1)$ is the predictive value of the plant model, $y(k-i),\ i=0,\ldots,H$, are the outputs of the plant, H is the prediction horizon, H_c is the control horizon, Δu is the incremented output of the controller, and e is the error between y and r.

2.4. Structure of fuzzy PI+D controller for predictive control

Fuzzy PD+I controller design is based on some previous work [20,21,23]. Here, we only outline the basic ideas (see Fig. 2).

As can been seen from Fig. 2, the transfer function of the linear PD+I controller is

$$U_{\mathrm{pid}}(s) = U_{\mathrm{pd}}(s) + U_{\mathrm{i}}(s),$$

where $U_{\mathrm{pd}}(s) = (K_{\mathrm{p}}^{\mathrm{c}} + s * K_{\mathrm{d}}^{\mathrm{c}})E(s)$, $U_{\mathrm{i}}(s) = \frac{K_{\mathrm{i}}^{\mathrm{c}}}{s}E(s)$, and $K_{\mathrm{p}}^{\mathrm{c}}$, $K_{\mathrm{i}}^{\mathrm{c}}$, and $K_{\mathrm{d}}^{\mathrm{c}}$ are constant PID control gains.

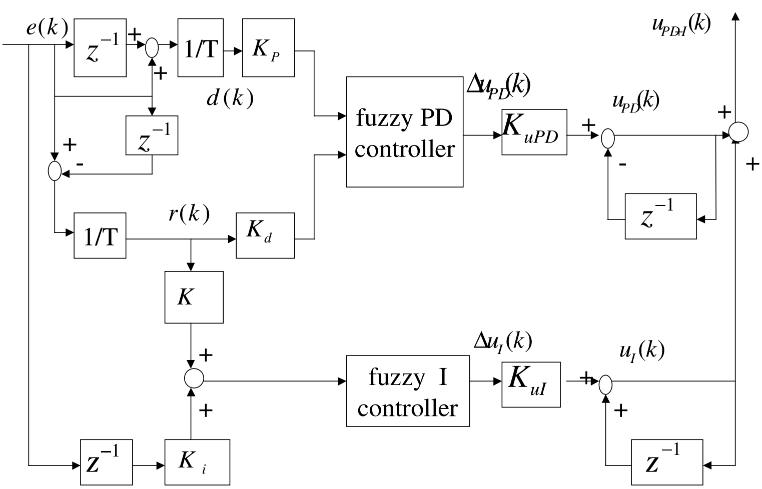

Fig. 2. The fuzzy PD+I controller.

By applying the bilinear transform,

$$s = \frac{2}{T}\frac{z-1}{z+1},$$

and letting

$$r(k) = \frac{e(k) - e(k-1)}{T},$$

$$d(k) = \frac{e(k) + e(k-1)}{T},$$

$$\Delta U_{\mathrm{pd}}(k) = \frac{U_{\mathrm{pd}}(k) + U_{\mathrm{pd}}(k-1)}{T},$$

$$\Delta U_{\mathrm{i}}(k) = \frac{U_{\mathrm{i}}(k) - U_{\mathrm{i}}(k-1)}{T},$$

we get

$$\Delta U_{\mathrm{pd}}(k) = K_{\mathrm{p}} * d(k) + K_{\mathrm{d}} * r(k),$$

$$\Delta U_{\mathrm{i}}(k) = K_{\mathrm{i}} * r(k) + K_{\mathrm{i}} * e(k-1),$$

$$U_{\mathrm{pd}}(k) = -U_{\mathrm{pd}}(k-1) + K_{U_{\mathrm{pd}}} * \Delta U_{\mathrm{pd}}(k),$$

$$U_{\mathrm{i}}(k) = U_{\mathrm{i}}(k-1) + K_{U_{\mathrm{i}}} * \Delta U_{\mathrm{i}}(k),$$

where $K_{U_{\mathrm{pd}}}$, $K_{U_{\mathrm{i}}}$, K_{p}, K_{i}, and K_{d} are the constant fuzzy control gains to be determined, $K_{\mathrm{p}} = K_{\mathrm{p}}^{\mathrm{c}}$, $K_{\mathrm{d}} = 2K_{\mathrm{d}}^{\mathrm{c}}/T$, $K_{\mathrm{i}} = K_{\mathrm{i}}^{\mathrm{c}}$, $K = K_{\mathrm{i}}^{\mathrm{c}}T/2$. Then, we let

$$U_{\mathrm{pid}}(k) = U_{\mathrm{pd}}(k) + U_{\mathrm{i}}(k). \tag{6}$$

Finally, we get the overall fuzzy PD+I controller, which is shown in Fig. 2. We note that the error here means the error of the optimal index J as compared to its reference.

For the fuzzy PD controller fuzzification, Fig. 3 shows its input membership functions and Fig. 4 shows its output membership functions. The input space is divided into 10 input-combination (IC) regions as is shown in Fig. 5, which will also be used for the fuzzy I controller later. The following four fuzzy control rules are used:

Rule 1: IF $d = d.n$ AND $r = r.n$ Then PD-output $= o.p$,
Rule 2: IF $d = d.n$ AND $r = r.p$ Then PD-output $= o.n$,
Rule 3: IF $d = d.p$ AND $r = r.n$ Then PD-output $= o.p$,
Rule 4: IF $d = d.p$ AND $r = r.p$ Then PD-output $= o.z$,

where $d.p$ means error positive, $d.n$ means error negative, $r.p$ means error rate positive, $r.n$ means error rate negative, $o.p$ means output positive, $o.n$ means output zero, $o.z$ means output negative, AND is the Zadeh fuzzy AND operation.

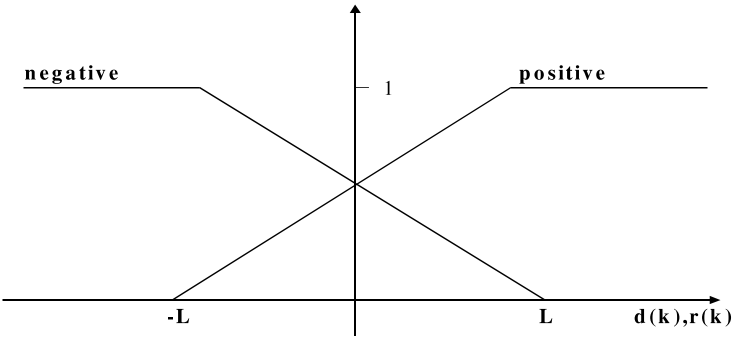

Fig. 3. Input membership functions for the fuzzy PD controller.

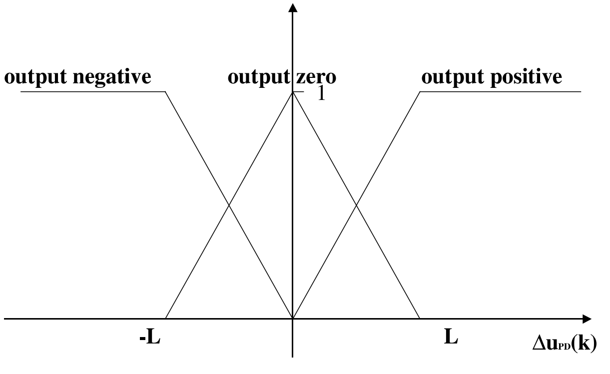

Fig. 4. Output membership functions for the fuzzy PD controller.

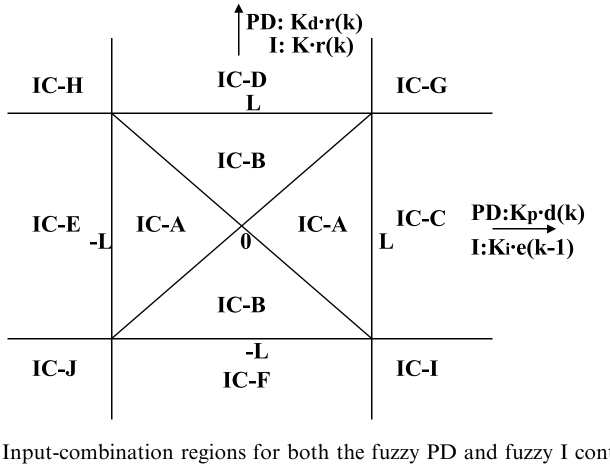

Fig. 5. Input-combination regions for both the fuzzy PD and fuzzy I controller.

The "center of mass" formula for defuzzification is used, which is

$$\Delta u(n) = \frac{\sum\{\text{input memebership value} \times \text{output membership value}\}}{\sum \text{output membership value}}. \tag{7}$$

By applying the output membership values $o.p = L$, $o.z = 0$, $o.n = -L$, and the following straight line formulas obtained from Fig. 5:

$$d.p = \frac{K_p d(k) + L}{2L},$$

$$d.n = \frac{-K_p d(k) + L}{2L},$$

$$r.p = \frac{K_d r(k) + L}{2L},$$

$$r.n = \frac{-K_d r(k) + L}{2L},$$

we get the following formulas for the 10 IC regions of the fuzzy PD controller:

$$\Delta u_{PD}(k) = \frac{L\left[K_p d(k) - K_d r(k)\right]}{2\left[2L - K_p|d(k)|\right]} \quad \text{in IC-A},$$

$$\Delta u_{PD}(k) = \frac{L[K_p d(k) - K_d r(k)]}{2[2L - K_d|r(k)|]} \quad \text{in IC-B},$$

$$\Delta u_{PD}(k) = \frac{-K_d r(k) + L}{2} \quad \text{in IC-C},$$

$$\Delta u_{PD}(k) = \frac{K_p d(k) - L}{2} \quad \text{in IC-D},$$

$$\Delta u_{PD}(k) = \frac{-K_d r(k) - L}{2} \quad \text{in IC-E},$$

$$\Delta u_{PD}(k) = \frac{K_p d(k) + L}{2} \quad \text{in IC-F},$$

$$\Delta u_{PD}(k) = 0 \quad \text{in IC-G and IC-J},$$

$$\Delta u_{PD}(k) = -L \quad \text{in IC-H},$$

$$\Delta u_{PD}(k) = L \quad \text{in IC-I}.$$

Similarly, for the fuzzy I controller, we use the same membership functions as those shown in Figs. 3 and 4, and use the following four fuzzy control rules:

Rule 5: IF $e(k-1) = e.n$ AND $r = r.n$ Then I-output $= o.n$,
Rule 6: IF $e(k-1) = e.n$ AND $r = r.p$ Then I-output $= o.z$,
Rule 7: IF $e(k-1) = e.p$ AND $r = r.n$ Then I-output $= o.z$,
Rule 8: IF $e(k-1) = e.p$ AND $r = r.p$ Then I-output $= o.p$.

By applying the output membership values $o.p = L$, $o.z = 0$, $o.n = -L$, and the following straight line formulas obtained from Fig. 5:

$$e.p = \frac{K_{\mathrm{i}}e(k-1)+L}{2L},$$
$$e.n = \frac{-K_{\mathrm{i}}e(k-1)+L}{2L},$$
$$r.p = \frac{Kr(k)+L}{2L},$$
$$r.n = \frac{-Kr(k)+L}{2L},$$

we get the following formulas for the 10 IC regions of the fuzzy I controller:

$$\Delta u_{\mathrm{I}}(k) = \frac{L[K_{\mathrm{i}}e(k-1)+Kr(k)]}{2[2L-K_{\mathrm{i}}|e(k-1)|]} \quad \text{in IC-A},$$
$$\Delta u_{\mathrm{I}}(k) = \frac{L[K_{\mathrm{i}}e(k-1)+Kr(k)]}{2[2L-K|r(k)|]} \quad \text{in IC-B},$$
$$\Delta u_{\mathrm{I}}(k) = \frac{Kr(k)+L}{2} \quad \text{in IC-C},$$
$$\Delta u_{\mathrm{I}}(k) = \frac{K_{\mathrm{i}}e(k-1)+L}{2} \quad \text{in IC-D},$$
$$\Delta u_{\mathrm{I}}(k) = \frac{Kr(k)-L}{2} \quad \text{in IC-E},$$
$$\Delta u_{\mathrm{I}}(k) = \frac{K_{\mathrm{i}}e(k-1)-L}{2} \quad \text{in IC-F},$$
$$\Delta u_{\mathrm{I}}(k) = 0 \quad \text{in IC-H and IC-I},$$
$$\Delta u_{\mathrm{I}}(k) = -L \quad \text{in IC-J},$$
$$\Delta u_{\mathrm{I}}(k) = L \quad \text{in IC-G}.$$

Therefore, totally seven parameters can be adjusted in this fuzzy PD+I controller, which means the predictive fuzzy PD+I controller has the same amount of adjustable parameters as the non-predictive controller. These parameters are K_{p}, K_{i}, K_{d}, $K_{u_{\mathrm{pd}}}$, $K_{u_{\mathrm{i}}}$, K, and L. We note that $K_{\mathrm{p}} = K_{\mathrm{p}}^{\mathrm{c}}$, $K_{\mathrm{d}} = 2 * (K_{\mathrm{d}}^{\mathrm{c}}/T)$, $K_{\mathrm{i}} = K_{\mathrm{i}}^{\mathrm{c}}$, $K = K_{\mathrm{i}}^{\mathrm{c}} * (T/2)$. Therefore, we usually adjust parameters $K_{\mathrm{p}}^{\mathrm{c}}$, $K_{\mathrm{i}}^{\mathrm{c}}$, $K_{\mathrm{d}}^{\mathrm{c}}$, $K_{u_{\mathrm{pd}}}$, and $K_{u_{\mathrm{i}}}$ when the sampling period T is fixed and the membership function parameter L is fixed.

3. Stability analysis of the new predictive fuzzy PID controller

Stability analysis is very important in the design of a controller. There are many stability analysis methods for fuzzy control system [24]: Small gain theorem, Lyapunov's second method, the graphical Lyaponov stability region analysis, etc. In addition, for analyzing the stability and performance of time-delay systems, there are classical stability analysis, stability analysis independent of time delay, stability analysis depending upon time delay, etc. In this section, we apply the Lyapunov second method to the stability analysis of the predictive fuzzy PD+I control system designed in Section 2.

3.1. Lyapunov second method

Consider the controlled system, which is now rewritten as the following compact form:

$$\begin{aligned} \dot{x} &= f(x) + F(x)\theta + g(x)u, \\ y &= h(x), \end{aligned} \tag{8}$$

where x is the state vector, θ is disturbance, u is controller, y is the system output and $f(x)$, $F(x)$, $g(x)$, $h(x)$ are nonlinear functions.

We want the output of system (8) to track the trajectory of the following reference system:

$$\begin{aligned} \dot{x}_{\mathrm{r}} &= f_{\mathrm{r}}(x_{\mathrm{r}}) + F_{\mathrm{r}}(x_{\mathrm{r}})\theta, \\ y_{\mathrm{r}} &= h(x_{\mathrm{r}}). \end{aligned} \tag{9}$$

The controller to be used is a predictive fuzzy PD+I controller designed in Section 2. Here, we should mention that the inputs of the fuzzy PD+I controller are the error and error rate signals from the optimal index J. The error of $J(k)$ is

$$e_J(k) = J(k) - J_{\mathrm{r}}(k).$$

The rate of error of $J(k)$ is

$$\dot{e}_J(k) = \dot{J}(k) - \dot{J}_{\mathrm{r}}(k) \approx J(k) - J(k-1) - \dot{J}_{\mathrm{r}}(k).$$

For simplicity of analysis, without loss of generality, we choose $J_{\mathrm{r}}(k) = 0$, so that $\dot{J}_{\mathrm{r}}(k) = 0$. Therefore, we have

$$\begin{aligned} e_J(k) &= J(k), \\ \dot{e}_J(k) &\approx J(k) - J(k-1). \end{aligned}$$

When the measurement is $y = h(x) = x$, $h_{\mathrm{r}}(x_{\mathrm{r}}) = x_{\mathrm{r}}$, i.e. $y = x$, we get the following formulas after subtracting (9) from (8):

$$\begin{aligned} e &= y - y_{\mathrm{r}} = x - x_{\mathrm{r}}, \\ \dot{e} &= \dot{y} - \dot{y}_{\mathrm{r}} = \dot{x} - \dot{x}_{\mathrm{r}} = f(x) - f_{\mathrm{r}}(x_{\mathrm{r}}) + (F(x) - F_{\mathrm{r}}(x_{\mathrm{r}}))\theta + g(x)u. \end{aligned}$$

If we represent

$$e_f = f(x) - f_r(x_r),$$
$$e_F = F(x) - F_r(x_r),$$

then

$$\dot{e} = e_f + e_F\theta + g(x)u.$$

If we choose a Lyapunov function to be

$$V = \frac{e_J^2}{2} > 0,$$

then we need to ensure $\dot{V} < 0$ uniformly, namely, we want to guarantee

$$\begin{aligned}\dot{V} = e_J\dot{e}_J &= J(k)[J(k) - J(k-1)] \\ &= \left[\sum_{i=-1}^{m} e^2(k-i) + \sum_{j=0}^{p} \Delta^2 u(k-j)\right] \\ &\quad \times \left[\sum_{i=-1}^{m} e^2(k-i) + \sum_{j=0}^{p} \Delta^2 u(k-j) - \sum_{i=-1}^{m} e^2(k-1-i) \right. \\ &\quad \left. - \sum_{j=0}^{p} \Delta^2 u(k-1-j)\right] \\ &= \left[\sum_{i=-1}^{m} e^2(k-i) + \sum_{j=0}^{p} \Delta^2 u(k-j)\right] \\ &\quad \times [e(k+1) - e(k-1-m) + \Delta u(k) - \Delta u(k-1-p)] < 0\end{aligned}$$

uniformly, where e is the tracking error of output y, and we have changed $H = m$, and $H_c = p$ for notational convenience.

Therefore, we can ensure $\dot{V}$ to be less than zero if at every step we can make sure

$$e(k) - e(k-1) < 0 \quad \text{and} \quad \Delta u(k) - \Delta u(k-1) < 0.$$

Now, we derive the conditions under which above condition are true for every k. First, we derive the conditions which ensure $e(k) - e(k-1) < 0$ at every step k. Notice that we have the following equations:

$$\begin{aligned} e(k) &= x(k) - x_r(k) = y(k) - y_r(k), \\ \dot{e}(k) &= e_f(k) + e_F(k)\theta(k) + g(k)u(k), \\ \dot{e}(k) &= e(k) - e(k-1), \\ e(k) &= e(k-1) + e_f(k) + e_F(k)\theta(k) + g(k)u(k). \end{aligned}$$

On the other hand,

$$u(k) = K_{u\mathrm{PD}}[K_{\mathrm{P}}d_J(k) + K_{\mathrm{d}}r_J(k)] + K_{u\mathrm{I}}[K_{\mathrm{i}}r_J(k) + K_{\mathrm{i}}e_J(k-1)] \\ - u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1)$$

or

$$\begin{aligned} u(k) = K_{u\mathrm{PD}}&\left[K_{\mathrm{P}}\frac{e_J(k)+e_J(k-1)}{T} + K_{\mathrm{d}}\frac{e_J(k)-e_J(k-1)}{T}\right] \\ &+ K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{e_J(k)-e_J(k-1)}{T} + K_{\mathrm{i}}e_J(k-1)\right] \\ &- u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1). \end{aligned} \tag{10}$$

When $J_{\mathrm{r}}(k) = 0$, i.e. $e_J(k) = J(k) - J_{\mathrm{r}}(k) = J(k)$, it becomes

$$\begin{aligned} u(k) = K_{u\mathrm{PD}}&\left[K_{\mathrm{P}}\frac{J(k)+J(k-1)}{T} + K_{\mathrm{d}}\frac{J(k)-J(k-1)}{T}\right] \\ &+ K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k)-J(k-1)}{T} + K_{\mathrm{i}}J(k-1)\right] \\ &- u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1). \end{aligned} \tag{11}$$

When we choose $g(k) = 1$, it gives

$$\begin{aligned} g(k)u(k) = K_{u\mathrm{PD}}&\left[K_{\mathrm{P}}\frac{J(k)+J(k-1)}{T} + K_{\mathrm{d}}\frac{J(k)-J(k-1)}{T}\right] \\ &+ K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k)-J(k-1)}{T} + K_{\mathrm{i}}J(k-1)\right] \\ &- u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1). \end{aligned} \tag{12}$$

Therefore,

$$\begin{aligned} e(k) - e(k-1) &= e_f(k) + e_F(k)\theta(k) + u(k) \\ &= e_f(k) + e_F(k)\theta(k) \\ &\quad + K_{u\mathrm{PD}}\left[K_{\mathrm{P}}\frac{J(k)+J(k-1)}{T} + K_{\mathrm{d}}\frac{J(k)-J(k-1)}{T}\right] \\ &\quad + K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k)-J(k-1)}{T} + K_{\mathrm{i}}J(k-1)\right] \\ &\quad - u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1). \end{aligned}$$

If there are constants $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and α_7, such that

$$
\begin{aligned}
&K_\mathrm{P}\frac{J(k)+J(k-1)}{T} < \alpha_1 < 0,\\
&K_\mathrm{d}\frac{J(k)-J(k-1)}{T} < \alpha_2 < 0,\\
&K_\mathrm{i}\left[\frac{J(k)-J(k-1)}{T} + J(k-1)\right] < \alpha_3 < 0,\\
&f(k) < \alpha_4,\\
&e_F(k)\theta(k) < \alpha_5,\\
&-u_\mathrm{PD}(k-1) < \alpha_6,\\
&u_\mathrm{i}(k-1) < \alpha_7,
\end{aligned}
$$

then we get

$$e(k) - e(k-1) < \alpha_4 + \alpha_5 + K_{u\mathrm{PD}}(\alpha_1 + \alpha_2) + K_{u\mathrm{I}}\alpha_3 + \alpha_6 + \alpha_7,$$

where α_6 and α_7 are upper bounds for the designed controllers, u_PD and u_I, respectively.

We want

$$e(k) - e(k-1) < \alpha_4 + \alpha_5 + K_{u\mathrm{PD}}(\alpha_1 + \alpha_2) + K_{u\mathrm{I}}\alpha_3 + \alpha_6 + \alpha_7 < 0.$$

Therefore, we may choose

$$
\begin{aligned}
&K_\mathrm{P} < \frac{\alpha_1 T_1}{J(k)+J(k-1)} = M_1,\\
&K_\mathrm{d} < \frac{\alpha_2 T}{J(k)-J(k-1)} = M_2,\\
&K_\mathrm{i} < \frac{\alpha_3 T}{[(J(k)-J(k-1))/T] + J(k-1)} = M_3,\\
&e_f(k) < \alpha_4,\\
&e_F(k)\theta(k) < \alpha_5,\\
&K_{u\mathrm{PD}} < -\frac{K_{u\mathrm{I}}\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7}{\alpha_1 + \alpha_2} = M_4,\\
&K_{u\mathrm{I}} < -\frac{K_{u\mathrm{PD}}(\alpha_1 + \alpha_2) + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7}{\alpha_3} = M_5,
\end{aligned}
$$

which together guarantee that for all k, uniformly,

$$e(k) - e(k-1) < 0.$$

We now derive the conditions under which $\Delta u(k) - \Delta u(k-1) < 0$ at every step k. We know that

$$\begin{aligned}
&\Delta u(k) - \Delta u(k-1)\\
&\quad = u(k) - u(k-1) - [u(k-1) - u(k-2)]\\
&\quad = u(k) - 2u(k-1) + u(k-2)\\
&\quad = K_{u\mathrm{PD}}\left[K_{\mathrm{P}}\frac{J(k)+J(k-1)}{T} + K_{\mathrm{d}}\frac{J(k)-J(k-1)}{T}\right]\\
&\qquad + K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k)-J(k-1)}{T} + K_{\mathrm{i}}J(k-1)\right] - u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1)\\
&\qquad - 2K_{u\mathrm{PD}}\left[K_{\mathrm{P}}\frac{J(k-1)+J(k-2)}{T} + K_{\mathrm{d}}\frac{J(k-1)-J(k-2)}{T}\right]\\
&\qquad - 2K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k-1)-J(k-2)}{T} + K_{\mathrm{i}}J(k-2)\right]\\
&\qquad + 2u_{\mathrm{PD}}(k-2) - 2u_{i}(k-2)\\
&\qquad + K_{u\mathrm{PD}}\left[K_{\mathrm{P}}\frac{J(k-2)+J(k-3)}{T} + K_{\mathrm{d}}\frac{J(k-2)-J(k-3)}{T}\right]\\
&\qquad + K_{u\mathrm{I}}\left[K_{\mathrm{i}}\frac{J(k-2)-J(k-3)}{T} + K_{\mathrm{i}}J(k-3)\right] - u_{\mathrm{PD}}(k-3) + u_{\mathrm{i}}(k-3)\\
&\quad = K_{u\mathrm{PD}}K_{\mathrm{P}}\frac{J(k)+J(k-1)-2J(k-1)-2J(k-2)+J(k-2)+J(k-3)}{T}\\
&\qquad + K_{u\mathrm{PD}}K_{\mathrm{d}}\frac{J(k)-J(k-1)-2J(k-1)+2J(k-2)+J(k-2)-J(k-3)}{T}\\
&\qquad + K_{u\mathrm{I}}K_{\mathrm{i}}\left[\frac{J(k)-J(k-1)-2J(k-1)+2J(k-2)+J(k-2)-J(k-3)}{T}\right.\\
&\qquad \left. + J(k-1) - 2J(k-2) + J(k-3)\right] - u_{\mathrm{PD}}(k-1) + u_{\mathrm{i}}(k-1)\\
&\qquad + 2u_{\mathrm{PD}}(k-2) - 2u_{\mathrm{i}}(k-2) - u_{\mathrm{PD}}(k-3) + u_{\mathrm{i}}(k-3)\\
&\quad = K_{u\mathrm{PD}}K_{\mathrm{P}}\frac{J(k)-J(k-1)-J(k-2)+J(k-3)}{T}\\
&\qquad + K_{u\mathrm{PD}}K_{\mathrm{d}}\frac{J(k)-3J(k-1)+3J(k-2)-J(k-3)}{T}\\
&\qquad + K_{u\mathrm{I}}K_{\mathrm{i}}\left[\frac{J(k)-3J(k-1)+3J(k-2)-J(k-3)}{T}\right.\\
&\qquad \left. + \ J(k-1) - 2J(k-2) + J(k-3)\right]\\
&\qquad - u_{\mathrm{PD}}(k-1) + 2u_{\mathrm{PD}}(k-2) - u_{\mathrm{PD}}(k-3)\\
&\qquad + u_{\mathrm{i}}(k-1) - 2u_{\mathrm{i}}(k-2) + u_{\mathrm{i}}(k-3).
\end{aligned} \tag{13}$$

Therefore, we may choose

$$K_{\rm P}\frac{J(k)-J(k-1)-J(k-2)+J(k-3)}{T}<\beta_1<0,$$
$$K_{\rm d}\frac{J(k)-3J(k-1)+3J(k-2)-J(k-3)}{T}<\beta_2<0,$$
$$K_{\rm i}\left[\frac{J(k)-3J(k-1)+3J(k-2)-J(k-3)}{T}\right.$$
$$\left.+J(k-1)-2J(k-2)+J(k-3)\right]<\beta_3<0,$$
$$-u_{\rm PD}(k-1)+2u_{\rm PD}(k-2)-u_{\rm PD}(k-3)<\beta_4,$$
$$u_{\rm i}(k-1)-2u_{\rm i}(k-2)+u_{\rm i}(k-3)<\beta_5,$$

which together guarantee that

$$\Delta u(k)-\Delta u(k-1)<K_{u{\rm PD}}(\beta_1+\beta_2)+K_{u{\rm I}}\beta_3+\beta_4+\beta_5,$$

where $\beta_1, \beta_2, \beta_3, \beta_4$, and β_5 are constants.

We want

$$\Delta u(k)-\Delta u(k-1)<K_{u{\rm PD}}(\beta_1+\beta_2)+K_{u{\rm I}}\beta_3+\beta_4+\beta_5<0.$$

Therefore, we may choose

$$K_{\rm P}<\frac{\beta_1T_1}{J(k)-J(k-1)-J(k-2)+J(k-3)}=N_1,$$
$$K_{\rm d}<\frac{\beta_2T}{J(k)-3J(k-1)+3J(k-2)-J(k-3)}=N_2,$$
$$K_{\rm i}<\beta_3/[(\{J(k)-3J(k-1)+3J(k-2)-J(k-3)\}/T)$$
$$+J(k-1)-2J(k-2)+J(k-3)]=N_3,$$

$$K_{u{\rm PD}}<\frac{K_{u{\rm I}}\beta_3+\beta_4+\beta_5}{\beta_1+\beta_2}=N_4,$$
$$K_{u{\rm I}}<\frac{K_{u{\rm PD}}(\beta_1+\beta_2)+\beta_4+\beta_5}{\beta_3}=N_5,$$
$$-u_{\rm PD}(k-1)+2u_{\rm PD}(k-2)-u_{\rm PD}(k-3)<\beta_4,$$
$$u_{\rm i}(k-1)-2u_{\rm i}(k-2)+u_{\rm i}(k-3)<\beta_5,$$

which ensure that

$$\Delta u(k)-\Delta u(k-1)<0.$$

Because we want both $e(k)-e(k-1)<0$ and $\Delta u(k)-\Delta u(k-1)<0$, we should choose $K_{\rm P}$, $K_{\rm I}$, $K_{\rm d}$, $K_{u{\rm PD}}$, and $K_{u{\rm I}}$ as follows:

$$
\begin{aligned}
&K_{\mathrm{P}} < \mathrm{Min}\{M_1, N_1\},\\
&K_{\mathrm{d}} < \mathrm{Min}\{M_2, N_2\},\\
&K_{\mathrm{i}} < \mathrm{Min}\{M_3, N_3\},\\
&K_{u\mathrm{PD}} < \mathrm{Min}\{M_4, N_4\},\\
&K_{u\mathrm{I}} < \mathrm{Min}\{M_5, N_5\}.
\end{aligned}
$$

When K_{P}, K_{I}, K_{d}, $K_{u\mathrm{PD}}$, and $K_{u\mathrm{I}}$ are chosen as above, we guarantee uniformly $\dot{V} < 0$. Therefore, the overall predictive fuzzy control system is asymptotically stable.

We summarize these results in the following Theorem 1.

Theorem 1. *If the designed predictive fuzzy PD+I controller satisfies the conditions listed below at every step, then the predictive fuzzy control system is asymptotically stable*:

$$
\begin{aligned}
&K_{\mathrm{P}} < \mathrm{Min}\{M_1, N_1\},\\
&K_{\mathrm{d}} < \mathrm{Min}\{M_2, N_2\},\\
&K_{\mathrm{i}} < \mathrm{Min}\{M_3, N_3\},\\
&K_{u\mathrm{PD}} < \mathrm{Min}\{M_4, N_4\},\\
&K_{\mathrm{I}} < \mathrm{Min}\{M_5, N_5\},\\
&e_f(k) < \alpha_4,\\
&e_F(k)\theta(k) < \alpha_5,\\
&-u_{\mathrm{PD}}(k-1) < \alpha_6,\\
&u_{\mathrm{i}}(k-1) < \alpha_7,\\
&-u_{\mathrm{PD}}(k-1) + 2u_{\mathrm{PD}}(k-2) - u_{\mathrm{PD}}(k-3) < \beta_4,\\
&u_{\mathrm{i}}(k-1) - 2u_i(k-2) + u_i(k-3) < \beta_5,
\end{aligned}
$$

where

$$
\begin{aligned}
&M_1 = \frac{\alpha_1 T_1}{J(k) + J(k-1)},\\
&N_1 = \frac{\beta_1 T_1}{J(k) - J(k-1) - J(k-2) + J(k-3)},\\
&M_2 = \frac{\alpha_2 T}{J(k) - J(k-1)},\\
&N_2 = \frac{\beta_2 T}{J(k) - 3J(k-1) + 3J(k-2) - J(k-3)},\\
&M_3 = \frac{\alpha_3 T}{[(J(k) - J(k-1))/T] + J(k-1)},
\end{aligned}
$$

$$N_3 = \beta_3/[((J(k) - 3J(k-1) + 3J(k-2) - J(k-3))/T) + J(k-1) - 2J(k-2) + J(k-3)],$$

$$M_4 = \frac{-(K_{uI}\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7)}{\alpha_1 + \alpha_2},$$

$$N_4 = \frac{K_{uI}\beta_3 + \beta_4 + \beta_5}{\beta_1 + \beta_2},$$

$$M_5 = \frac{-[K_{uPD}(\alpha_1 + \alpha_2) + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7]}{\alpha_3},$$

$$N_5 = \frac{K_{uPD}(\beta_1 + \beta_2) + \beta_4 + \beta_5}{\beta_3}.$$

4. Simulation results

In this section, we present some simulation results to show the effectiveness and special features of the proposed control method. We call the proposed method optimal J based predictive Fuzzy PD+I control method, which is shown in Fig. 1. We compare it with several other closely related control methods for controlling several linear or nonlinear, certain or uncertain systems.

We focus on comparison of the new method with two particularly important and comparable methods. First, we compare it with the non-predictive fuzzy PD+I control method to show the significant feature of tracking optimal index J and the influences of the predictive model identification. Second, we compare it with a Smith-type predictive fuzzy PD+I control method to show the special feature of tracking optimal index J. Both methods have different influences of model identification errors.

The non-predictive fuzzy PD+I control method is described in [23] and is shown in Fig. 6. A Smith-type fuzzy predictive control method is shown in

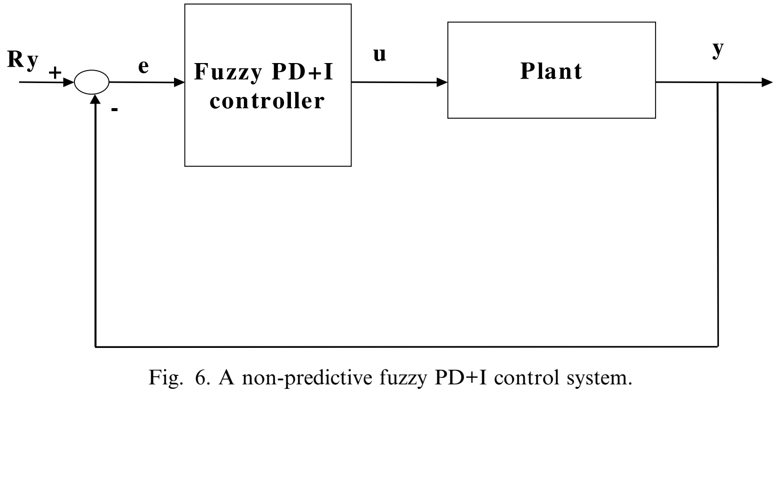

Fig. 6. A non-predictive fuzzy PD+I control system.

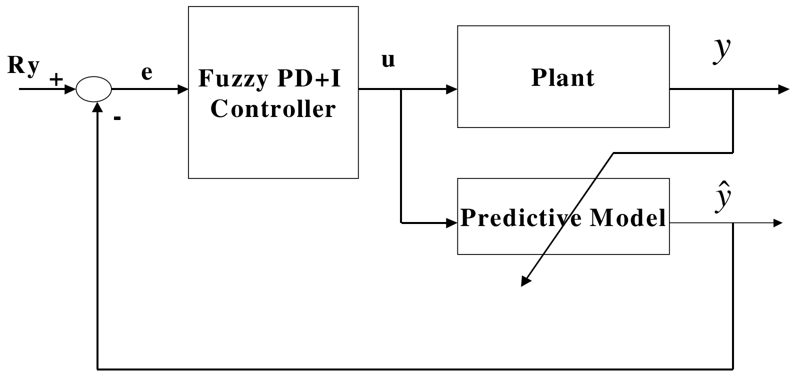

Fig. 7. A Smith-type predictive fuzzy PD+I control system.

Fig. 7. It is composed of a Smith-type predictive model and a fuzzy PD+I controller.

We have compared these control methods for controlling the following seven systems:
- A nonlinear system with time delay.
- A linear uncertain system with time delay.
- The Hénon chaotic system.
- A linear system without time delay.
- A linear system with time delay.
- A nonlinear system without time delay.
- A nonlinear uncertain system with time delay.

Due to the limitation of space, we only present the comparison results of the first three systems in this paper.

4.1. Nonlinear system with time delay

Consider a nonlinear system with time delay:

$$y(k) = -y(k-1) + 0.5 * y^2(k-1) + u(k-6).$$

(1) *Results of the optimal J based predictive fuzzy PD+I controller*:
We choose the optimal index as

$$J(k) = \sum_{i=-1}^{7} e^2(k-i) + \sum_{j=0}^{7} (\Delta u(k-j))^2.$$

The parameters for the predictive fuzzy PD+I controller are optimized at: $L = 100$, $T = 0.1$, $K_p^c = 0.35$, $K_d^c = 0.000001$, $K_i^c = 0.75$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 0.01 * T = 0.001$, $K_{uI} = 1 * T = 0.1$.

The predictive model is

$$\hat{y}(k+1) = a_1 * y(k) + a_2 * y(k-1) + b_0 U_{\text{pid}}(k) + b_1 U_{\text{pid}}(k) + \cdots + b_6 U_{\text{pid}}(k-5),$$

where $a_1, a_2, b_0, b_1, \ldots, b_6$ are identified on-line. The simulation result of the above nonlinear system controlled by the predictive fuzzy PD+I controller without identification is shown in Fig. 8. Here we need mention that the J in this figure and the following figures represents $\text{sign}(e(k))J(k)$. The total output error is 46.6534.

(2) *Results of the non-predictive fuzzy PD+I controller*:

We choose the optimized parameters for the non-predictive fuzzy PD+I controller at $L = 10$, $T = 0.1$, $K_p^c = 0.175$, $K_d^c = 0.000003$, $K_i^c = 2.9$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 1 * T = 0.1$, $K_{uI} = 1 * T = 0.1$. The result of the same nonlinear system with time delay controlled by the non-predictive fuzzy PD+I controller without identification is shown in Fig. 9.The total error between the plant output and the output reference is 40.3817.

(3) *Results of the Smith-type predictive fuzzy PD+I controller*:

The optimal parameters for the Smith-type predictive fuzzy PD+I controller are set at: $L = 10$, $T = 0.1$, $K_p^c = 0.175$, $K_d^c = 0.000003$, $K_i^c = 2.9$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 1 * T = 0.1$, $K_{uI} = 1 * T = 0.1$.

The predictive model is

$$\hat{y}(k+1) = a_1 * y(k) + a_2 * y(k-1) + b_0 U_{\text{pid}}(k) + b_1 U_{\text{pid}}(k) + \cdots + b_6 U_{\text{pid}}(k-5),$$

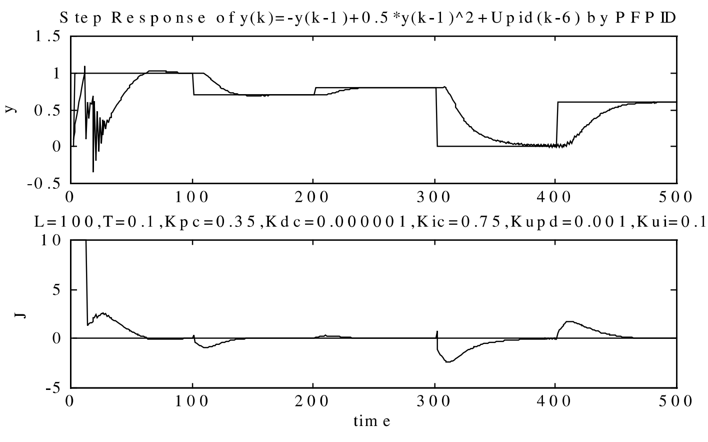

Fig. 8. The result of a nonlinear system with time delay controlled by the predictive fuzzy PD+I controller without model identification.

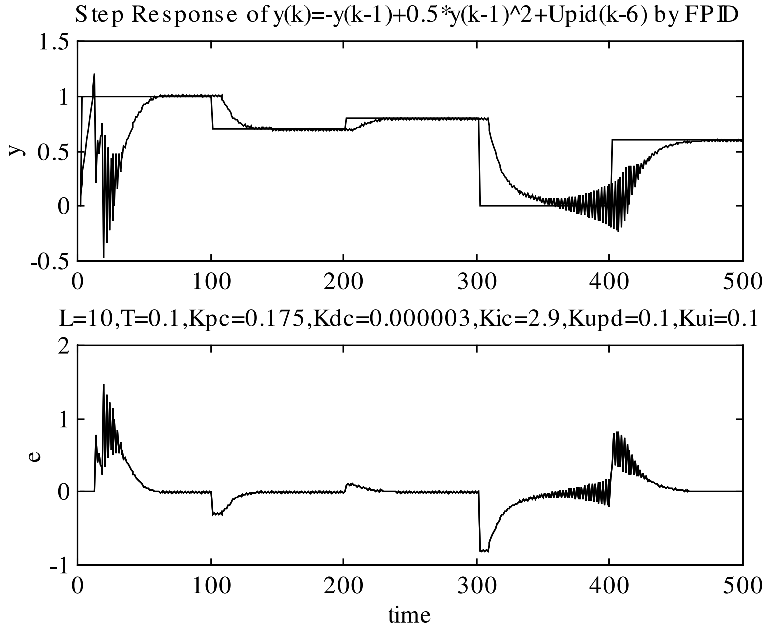

Fig. 9. The result of a nonlinear system with time delay controlled by the non-predictive fuzzy PD+I controller without model identification.

where $a_1, a_2, b_0, b_1, \ldots b_6$ are identified on-line. The simulation result of the same nonlinear system controlled by the Smith-type predictive fuzzy PD+I controller with identification is shown in Fig. 10. It shows that the control system is unstable.

For comparison purpose, we present Table 1 which shows the system output errors created by these three different control methods. These results are obtained for the different optimized PD+I parameters. It shows that the predictive fuzzy PD+I control method has better performance than the non-predictive fuzzy PD+I control method for the nonlinear system with time delay when both methods do not use model identification although the former has a slight bigger sum of error than the later. It also shows that both the predictive fuzzy PD+I control method and the Smith-type predictive fuzzy control method fail because of the model identification process. If we compare Figs. 8–10, we can see that the predictive fuzzy PD+I control method has better performance than the non-predictive fuzzy PD+I control method when the control system is the nonlinear system with time delay. Therefore, the predictive fuzzy PD+I control method has more advantages over the non-predictive fuzzy PD+I control method when the system under control is a nonlinear system with time delay.

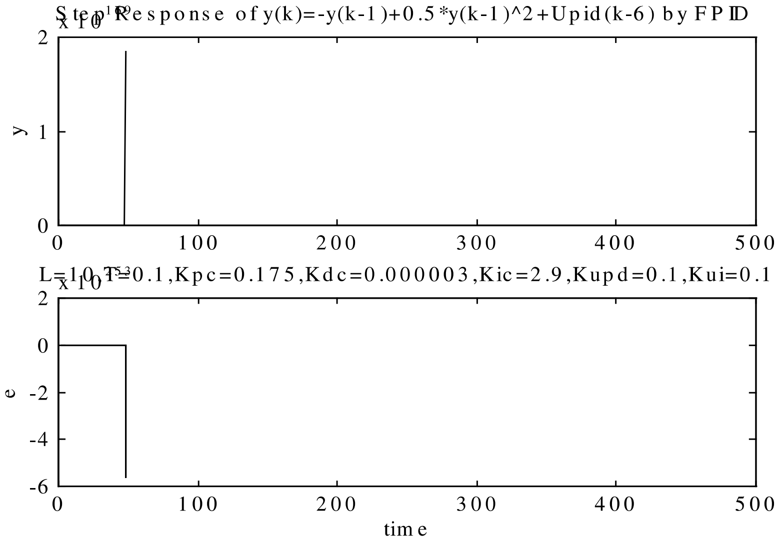

Fig. 10. The result of the nonlinear system with time delay controlled by the Smith-type predictive fuzzy PD+I controller with model identification.

Table 1
The errors of different control methods for a nonlinear system with time delay

Control method	System output error
Predictive fuzzy PD+I control	46.6534 (without model identification)
	Infinity (with model identification)
Non-predictive fuzzy PD+I control	40.3817 (without model identification)
Smith-type predictive fuzzy PD+I control	Infinity (with model identification)

4.2. Linear uncertain system with time delay

The linear uncertain system with time delay is

$$y(k) = (0.9z^{-1} - 0.18z^{-2})y(k) + z^{-6}(0.5 + 0.25z^{-1})u(k) + 0.9 * \mathrm{rand}(k).$$

(1) *Results of the optimal J based predictive fuzzy PD+I controller*:

We set the following parameters for the predictive fuzzy PD+I controller: $L = 462$, $T = 0.1$, $K_{\mathrm{p}}^{\mathrm{c}} = 0.6$, $K_{\mathrm{d}}^{\mathrm{c}} = 0.001$, $K_{\mathrm{i}}^{\mathrm{c}} = 0.25$, $K_{\mathrm{p}} = K_{\mathrm{p}}^{\mathrm{c}}$, $K_{\mathrm{d}} = 2 * (K_{\mathrm{d}}^{\mathrm{c}}/T)$, $K_{\mathrm{i}} = K_{\mathrm{i}}^{\mathrm{c}}$, $K = K_{\mathrm{i}}^{\mathrm{c}} * (T/2)$, $K_{u\mathrm{PD}} = 0.1 * T = 0.01$, $K_{u\mathrm{I}} = 1 * T = 0.1$. The predictive model is

$$\hat{y}(k+1) = a_1 y(k) + a_2 y(k-1) + b_0 U_{\mathrm{pid}}(k) + b_1 U_{\mathrm{pid}}(k) + \cdots + b_7 U_{\mathrm{pid}}(k-6) + 0.9 * \mathrm{rand},$$

where $a_1, a_2, b_0, b_1, \ldots, b_7$ are identified on-line.

The optimal index is chosen as

$$J(k) = \sum_{i=-1}^{9} e^2(k-i) + \sum_{j=0}^{9} (\Delta u(k-j))^2,$$

where, again, $m = p = 9$ is used to have a long enough time horizon, so that the controller is able to handle the uncertainty.

We present two results. One is with predictive model identification and the other without. Both results use the same setting shown above. The result of the linear uncertain time-delay system controlled by a predictive fuzzy PD+I controller without predictive model identification is shown in Fig. 11. The total error of system output is 802.8372. The result of the same system controlled by the predictive fuzzy PD+I controller with predictive model identification is shown in Fig. 12. The total error of system output is 1909.00.

(2) *Results of the non-predictive fuzzy PD+I controller*:

Optimized parameters for the fuzzy PD+I controller are chosen as: $L = 462$, $T = 0.1$, $K_p^c = 0.01$, $K_d^c = 0.001$, $K_i^c = 0.8$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPU} = 1 * T = 0.1$, $K_{uI} = 1 * T = 0.1$. The result is shown in Fig. 13. The total error of system output is 882.2258.

(3) *Results of the Smith-type predictive fuzzy PD+I controller*:

We use the optimized parameters for the Smith-type predictive fuzzy PD+I controller, i.e. $L = 462$, $T = 0.1$, $K_p^c = 0.01$, $K_d^c = 0.001$, $K_i^c = 0.8$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{U_{PD}} = 1 * T = 0.1$, $K_{Ui} = 1 * T = 0.1$. The predictive model is

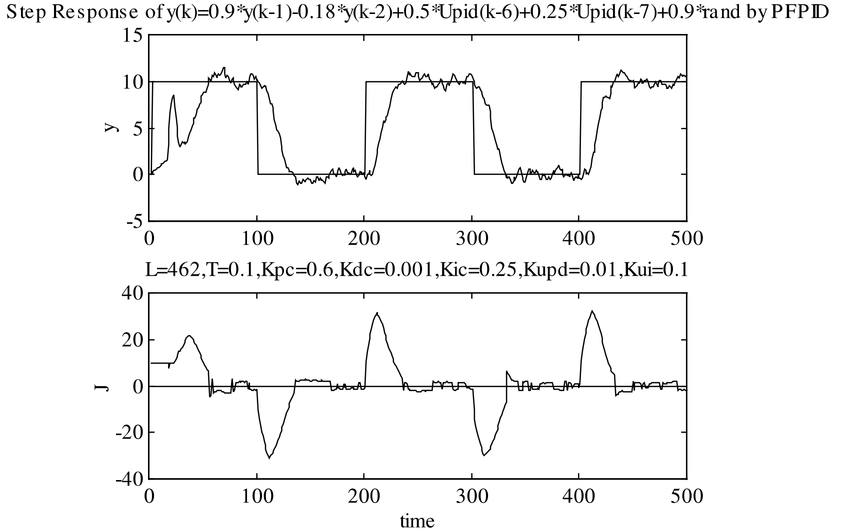

Fig. 11. The result of the disturbed linear time-delay system controlled by the predictive fuzzy PD+I controller without model identification.

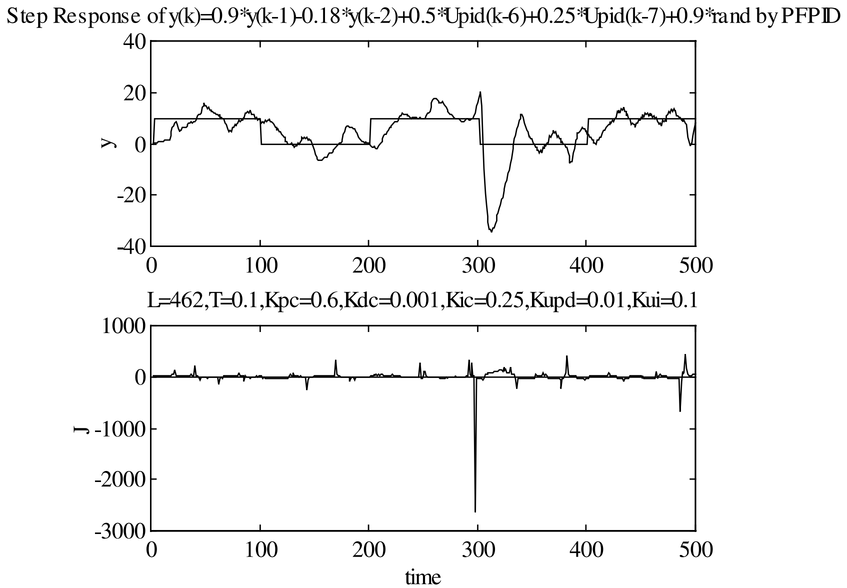

Fig. 12. The result of the disturbed linear time-delay system controlled by the predictive fuzzy PD+I controller with model identification.

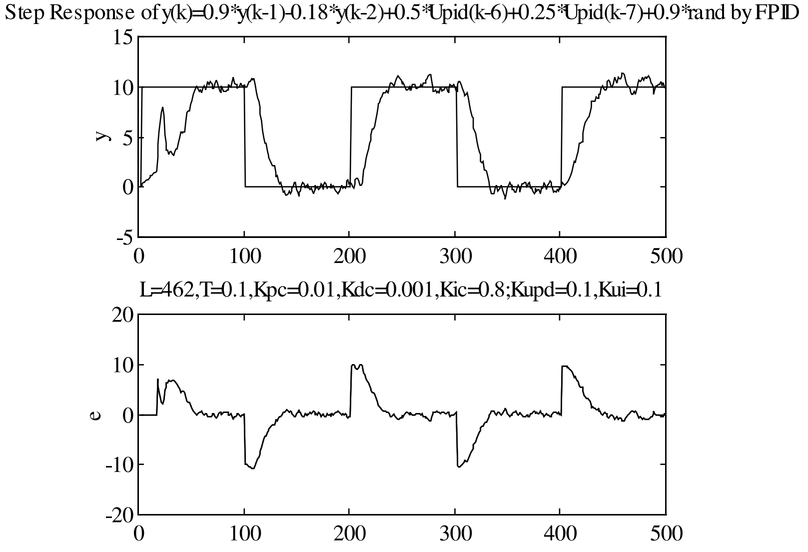

Fig. 13. The result of the disturbed time-delay linear system controlled by the fuzzy PD+I controller without model identification.

$$\hat{y}(k+1) = a_1 y(k) + a_2 y(k-1) + b_0 U_{\text{pid}}(k) + b_1 U_{\text{pid}}(k) + \cdots + b_7 U_{\text{pid}}(k-6), \tag{14}$$

where $a_1, a_2, b_0, b_1, \ldots, b_7$ are identified on-line.

The result of the uncertain time-delay linear system controlled by the Smith-type predictive fuzzy PD+I controller with model identification is shown in Fig. 14. The total error of system output is 6270; the total error of predictive system output is 24 869.

Now, for comparison, we list the errors in Table 2. From this table and Figs. 11–14, we can see that the predictive fuzzy PD+I controller is better than the non-predictive fuzzy PD+I controller and the Smith-type predictive PD+I controller, when controlling the uncertain time-delay linear system.

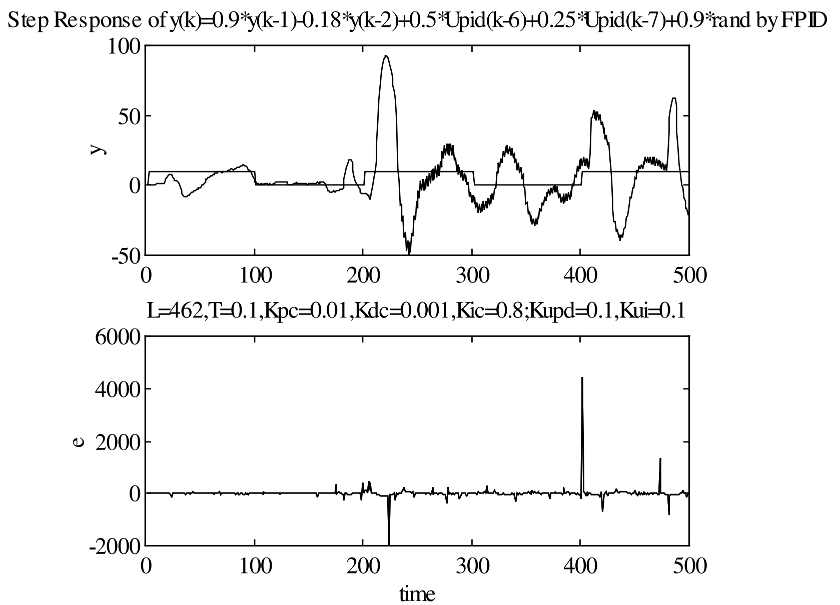

Fig. 14. The result of the disturbed linear time-delay system controlled by the Smith-type predictive fuzzy PD+I controller with model identification.

Table 2
The errors of different control methods for an uncertain time-delay linear system

Control method	System output error
Predictive fuzzy PD+I control	802.8372 (without model identification)
	1909 (with model identification)
Non-predictive fuzzy PD+I control	882.2558 (without model identification)
Smith-type predictive fuzzy PD+I control	24 869 (with model identification)

4.3. Chaotic system

In this section, we present the results of controlling the Hènon chaotic system by using the different control methods. The Hènon chaotic system is

$$y(k) = 1.3 - 1.3 * y^2(k-2) - 0.065y(k-3) + u(k-1).$$

The objective of controlling this chaotic system is to stabilize it to the zero equilibrium point and 0.5 equilibrium point.

(1) *Results of the optimal J based predictive fuzzy PD+I controller*:

To stabilize this chaotic system, we choose the optimal index as

$$J(k) = \sum_{i=-1}^{3} e^2(k-i) + \sum_{j=0}^{3} (\Delta u(k-j))^2.$$

The parameters for the predictive fuzzy PD+I controller are chosen as: $L = 462$, $T = 0.1$, $K_p^c = 0.0005$, $K_d^c = 0.00001$, $K_i^c = 2.955$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 0.01 * T = 0.001$, $K_{uI} = 0.2 * T = 0.02$.

The predictive model is

$$\hat{y}(k+1) = a_0 + a_1 y(k) + \cdots + a_4 y(k-3) + b_0 U_{pid}(k) + b_1 U_{pid}(k-1),$$

where $a_0, a_1, \ldots, a_4, b_0$, and b_1 are identified on-line.

The simulation result of this chaotic system controlled by the predictive fuzzy PD+I controller without model identification is shown in Fig. 15. The total error of system output is 44.7917. The simulation result of the chaotic system controlled by the predictive fuzzy PD+I controller with model identification is shown in Fig. 16. It shows that the closed-loop system is unstable.

(2) *Results of the non-predictive fuzzy PD+I controller*:

To control this Hènon chaotic system, the parameters for the non-predictive fuzzy PD+I controller are optimized as $L = 462$, $T = 0.1$, $K_p^c = 0.0007$, $K_d^c = 0.0001$, $K_i^c = 1.55$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 1 * T = 0.1$, and $K_{uI} = 1 * T = 0.1$. The simulation result of the Hènon chaotic system controlled by the non-predictive fuzzy PD+I controller without identification is shown in Fig. 17. The total error of system output is 76.2353.

(3) *Results of the Smith-type predictive fuzzy PD+I controller*:

Here, the optimized parameters for the Smith-type predictive fuzzy PD+I controller are chosen as: $L = 462$, $T = 0.1$, $K_p^c = 0.0007$, $K_d^c = 0.0001$, $K_i^c = 1.55$, $K_p = K_p^c$, $K_d = 2 * (K_d^c/T)$, $K_i = K_i^c$, $K = K_i^c * (T/2)$, $K_{uPD} = 0.002 * T = 0.0002$, $K_{uI} = 0.2 * T = 0.02$.

The predictive model is

$$\hat{y}(k+1) = a_0 + a_1 y(k) + \cdots + a_4 y(k-3) + b_0 U_{pid}(k) + b_1 U_{pid}(k-1),$$

where $a_0, a_1, \ldots, a_4, b_0$, and b_1 are identified on-line.

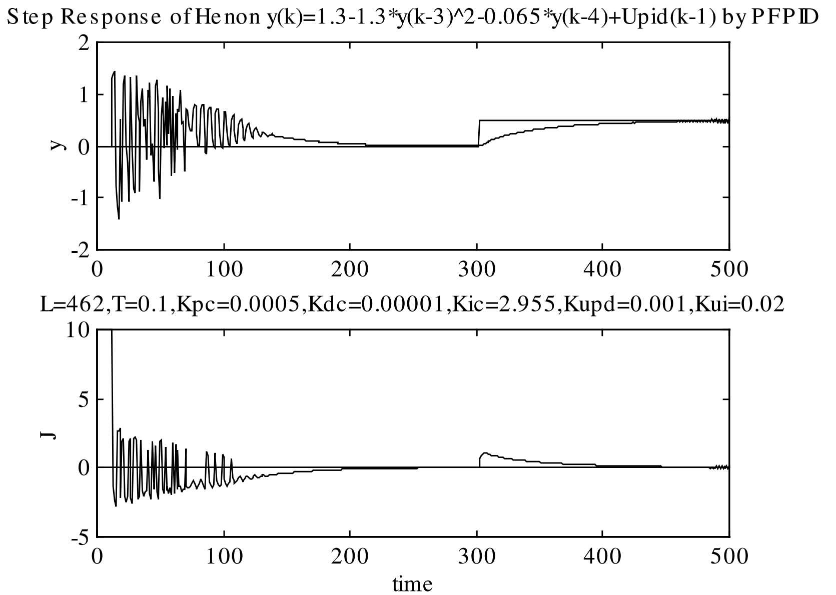

Fig. 15. The chaotic system controlled by the predictive fuzzy PD+I controller without model identification.

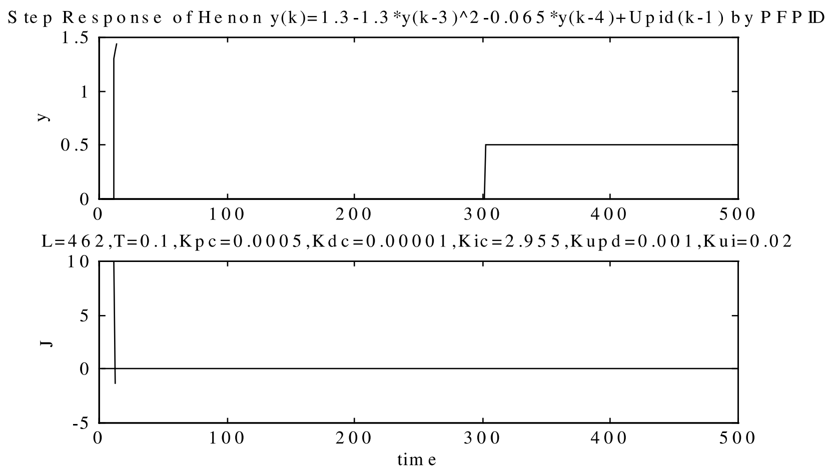

Fig. 16. The chaotic system controlled by the predictive fuzzy PD+I controller with model identification.

The simulation result of the Hènon chaotic system controlled by the Smith-type fuzzy PD+I controller with identification is shown in Fig. 18. The control system is unstable.

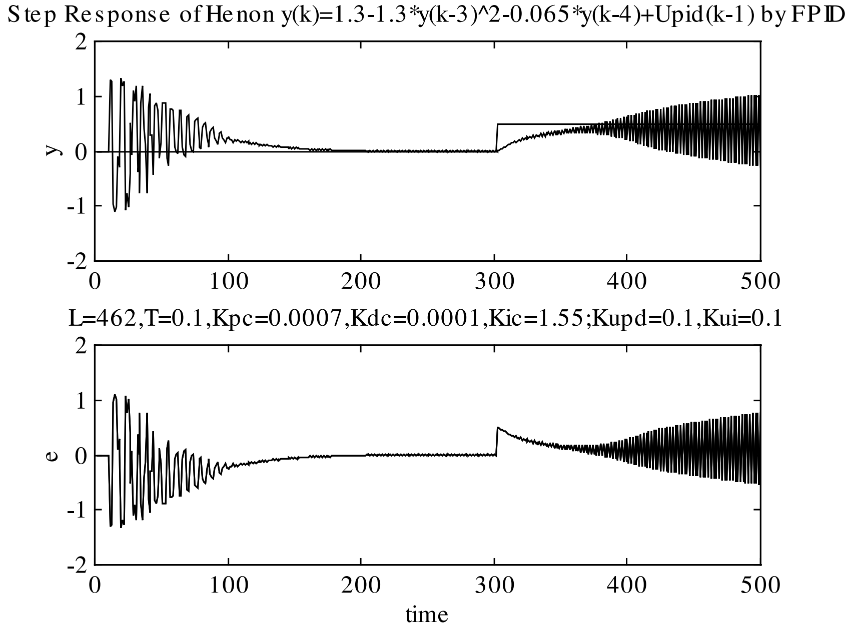

Fig. 17. Hènon chaotic system controlled by the fuzzy PD+I controller without model identification.

To better compare these control methods, we present Table 3 which shows the errors created by these three different control methods. The table shows that the predictive fuzzy PD+I control method has smaller errors than the non-predictive fuzzy PD+I control method when both methods do not use model identification. It also shows that both the predictive fuzzy PD+I control method and the Smith-type predictive fuzzy control method fail because of the model identification process. If we compare Figs. 15–18, we can see that the predictive fuzzy PD+I control method has better performance than the non-predictive fuzzy PD+I control method.

5. Conclusions

In this paper, we have developed a new predictive fuzzy PID controller, and have carried out a rigorous Lyapunov asymptotic stability analysis for the overall control system. We have verified its effectiveness. We have also verified the performance and system stability. We use the PID gains K_p, K_I, and K_D to achieve good control performance and use the optimal cost function J to

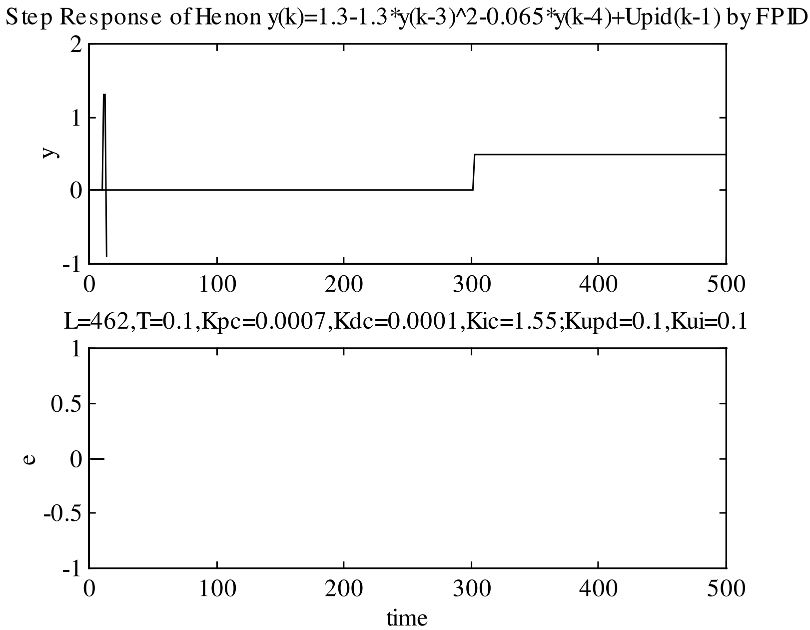

Fig. 18. The Hènon chaos system controlled by the Smith-type fuzzy PD+I controller with model identification.

Table 3
The errors of different control methods for the Hènon chaotic system

Control method	System output error
Predictive fuzzy PD+I control	44.7917 (without model identification)
	Infinity (with model identification)
Non-predictive fuzzy PD+I control	76.2353 (without model identification)
Smith-type predictive fuzzy PD+I control	Infinity (with model identification)

handle the constrained conditions and system stability. We have also compared the new predictive fuzzy controller with other closely related and comparable predictive control methods in simulation.

All the simulation results indicate that the proposed predictive fuzzy PD+I controller is superior to both the fuzzy PD+I controller and the Smith-type predictive fuzzy PD+I controller. The new controller has faster response than the conventional GPC, and does not require a precise model. The new controller embeds the fuzzy PID controllers with a predictive feature. It is suitable for controlling uncertain linear and nonlinear time-delay systems with guaranteed stability.

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