TITO Mamdani Fuzzy PI/PD Controllersas Nonlinear, Variable Gain PI/PD controllers

Hao Ying

Abstract

In this paper, analytical structures of TITO (two-input two-output) Mamdani fuzzy PI/PD controllers are investigated with respect to conventional PI/PD control and variable gain control. Components of the fuzzy controllers include two input fuzzy sets for each input variable, five singleton output fuzzy sets for each output variable, 16 fuzzy rules, product AND fuzzy logic operator, the Mamdani minimum inference method and the centroid defuzzifier. Each output of the fuzzy controller is proved to be the sum of two nonlinear PI (or PD) controllers with variable gains that continuously change with the outputs of the TITO system under control. The characteristics of the fuzzy controller are analyzed in relation to those of the corresponding TITO linear PI/PD control. Finally, it is theoretically proved that the fuzzy and linear PI/PD control systems have the same local stability at the eqlibrium points.

1.Introduction

Revealing explicit structure of fuzzy controllers [8,20] is foremost important because it can provide insightful information about what a fuzzy controller is, how it works, and how it relates to and differs from a conventional controller. A fuzzy controller is not fuzzy anymore (i.e., not a black-box controller anymore) once its explicit structure is disclosed. It becomes just a conventional nonlinear controller. As a result, the structure provides a platform on which the well-developed controls and systems theories can be utilized to precisely analyze and design fuzzy controllers as well as fuzzy control systems.

Corresponding author: Hao Ying is with Department of Electrical and Computer Engineering
Wayne State University
Detroit, Michigan 48202, USA
hying@ieee.edu

The analytical structures of diverse SISO (single-input single-output) Mamdani fuzzy controllers have been studied by many groups (e.g., [4,6,7,9-12,15-17]), including ours. We have proved that various SISO Mamdani PI/PD controllers are nonlinear PI/PD controllers with variable gains (e.g., [21-24,26]).

MIMO (multiple-input multiple-output) fuzzy controllers are desirable in many situations. However, the number of existing MIMO fuzzy control applications is rather small. Notable applications include the flexible wing aircraft control [1] and the steam generating unit regulation [13]. MIMO theoretical results are scarce, and most of the few results in existence are on the traditional fuzzy-relation-based MIMO fuzzy controllers (e.g., [3,5]). Linguistic decoupling control of fuzzy multivariable processes was attempted [18,19]. More recently, we derived the anlaytical structure of the TITO (two-input two-output) Mamdani fuzzy controller with linear fuzzy rules [25].

In this paper, we extend our SISO fuzzy PI/PD results to the TITO cases. Specifically, we will derive the analytical structures of TITO Mamdani fuzzy PI/PD controllers in terms of PI/PD controllers. Given the dominance of conventional PID control in industrial applications [2], it is significant both in theory and in practice, if a controller can be found that is capable of outperforming the PID controller. We will then analyze the characteristics of the resultant structures. Finally, we will look into local stability of the fuzzy control systems at the equilibrium points.

We focus on local stability instead of global stability because, in our opinion, the two types of stability are equally important; one cannot replace the other as each type has its distinctive merits and drawbacks. A local stability condition is usually a necessary and sufficient condition. On the other hand, in most cases, a global stability condition, especially if it is for nonlinear systems, is a sufficient condition. Necessary one is uncommon. Except for linear systems, it is rare that a global stability condition is a necessary and sufficient condition. The most widely used and effective methodology for global stability determination was developed by Lyapunov, which requires a Lyapunov function to be constructed for the control system involved. For any specific system, more than one

Lyapunov function exist. Regardless of the methodologies, their foremost assumption for establishing a global stability condition, sufficient or necessary, is the availability of the analytical expressions of both the controller and the system. This assumption is critical: Global stability analysis is impossible without it. This assumption, however, is rather unrealistic and impractical in many situations because the precise mathematical model of the nonlinear system involved is often unavailable (It makes little sense to use a fuzzy controller to control a linear system).

Even if in the cases where the accurate model is available, properly determining global stability for fuzzy control systems can still be difficult. Lyapunov functions are system-dependent, and its construction is more an art than science. There does not exist a general method to automatically construct an appropriate Lyapunov function for any given nonlinear system. Due to fuzzy controllers' structural complexity and peculiarity (that is, nonlinear and time-varying), global stability of fuzzy control systems can only be judged on case by case basis at best.

2.Configurations of the TITO Mamdani fuzzy PI/PD controllers

2.1 Mamdani Fuzzy PI Controller

The TITO Mamdani fuzzy PI controller in this study uses the following input variables:

$$x_1(k) = k_1 \cdot e_1(k) = k_1(SP_1(k) - y_1(k)),$$

$$x_2(k) = k_2 \cdot \Delta e_1(k) = k_2(y_1(k-1) - y_1(k)),$$

$$x_3(k) = k_3 \cdot e_2(k) = k_3(SP_2(k) - y_2(k)),$$

$$x_4(k) = k_4 \cdot \Delta e_2(k) = k_4(y_2(k-1) - y_2(k)),$$

where $y_1(k)$ and $y_2(k)$ are coupled outputs of the TITO system under control with respective setpoints of $SP_1(k)$ and $SP_2(k)$. Here, $e_1(k)$ and $e_2(k)$ denote errors of the system outputs whereas $\Delta e_1(k)$ and $\Delta e_2(k)$ designate change of errors (rates, for short). k_i's (i = 1,…,4) are scaling factors, and k and k-1 represent current and previous sampling times, respectively.

The scaled input variables are fuzzified by two input fuzzy sets, shown in Fig. 1. They are called "Positive" and "Negative," and are represented respectively by X_i^1 and X_i^{-1} for $x_i(k)$. Without lossing generaility, we only study in this paper the cases where $|x_i(k)| \le L$. L is a design parameter, and -L and L are two turning points of the membership functions in Fig. 1. It is evident that the membership functions of X_i^1 and X_i^{-1} are respectively

$$\mu_1(x_i) = \frac{L + x_i(k)}{2L}$$

and

$$\mu_{-1}(x_i) = \frac{L - x_i(k)}{2L}, \qquad (1)$$

and

$$\mu_1(x_i) + \mu_{-1}(x_i) = 1, \qquad \text{for } i = 1,\ldots,4.$$

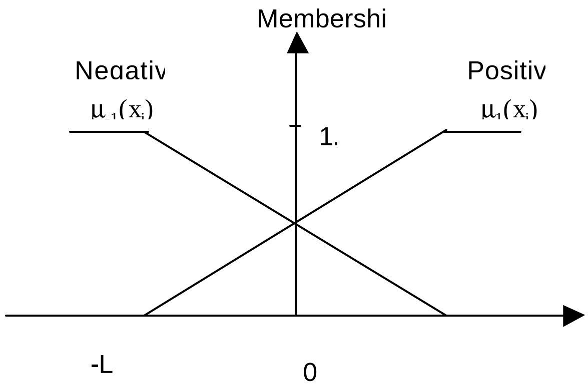

Figure 1. Illustrative definition of the triangular input fuzzy sets.

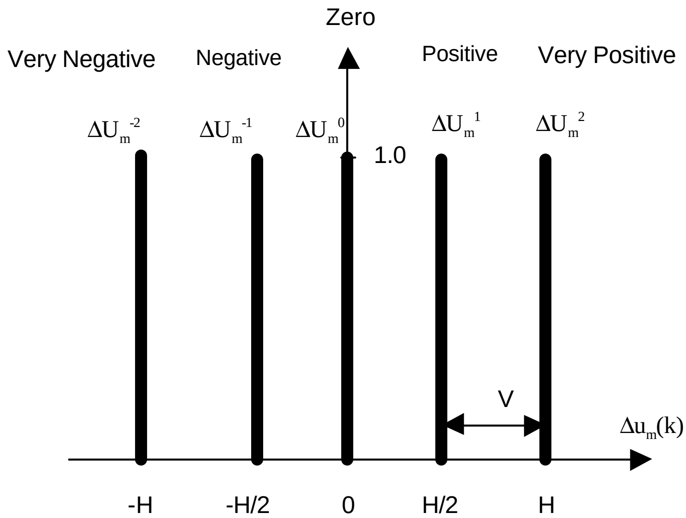

Figure 2. Illustrative definition of the singleton output fuzzy sets.

Five singleton output fuzzy sets, defined on [-H, H] as shown in Fig. 2, are used for two incremental output variables of the fuzzy controller, namely $\Delta u_1(k)$ and $\Delta u_2(k)$. They are expressed as ΔU_m^2, ΔU_m^1, ΔU_m^0, ΔU_m^{-1} and ΔU_m^{-2} (m = 1, 2), which may be called "Very Positive," "Positive," "Zero," "Negative" and "Very Negative," respectively. The membership of $\Delta U_m^{j_m}$ is 1 at $j_m V$, where j_m = -2, -1, 0, 1, 2 and V = H/2, and is 0 anywhere else.

Because there are total 2^4 different combinations for the four scaled input variables, 16 fuzzy control rules are needed. They are listed in Table 1. The rules use the following notation:

IF $x_1(k)=X_1^{I_1}$ AND $x_2(k)=X_2^{I_2}$ AND $x_3(k)=X_3^{I_3}$ AND $x_4(k)=X_4^{I_4}$

THEN $\Delta u_1(k)=\Delta U_1^{J_1}$, $\Delta u_2(k)=\Delta U_2^{J_2}$

where the input fuzzy sets and output fuzzy sets are connected by the following relationships:

$$J_1=\frac{I_1+I_2}{2}+\delta_1\frac{I_3+I_4}{2},$$

$$J_2=\delta_2\frac{I_1+I_2}{2}+\frac{I_3+I_4}{2}.$$

Here, δ_2 (δ_1) should be -1 if increasing $y_1(k)$ ($y_2(k)$) causes increase of $y_2(k)$ ($y_1(k)$). Otherwise, δ_2 (δ_1) should be +1. δ_1 (δ_2) should be zero if $y_1(k)$ and $y_2(k)$ do not couple with each other.

To evaluate ANDs in the rule antecedents, product AND fuzzy logic operator is used. And, the Mamdani minimum inferene method is used for the fuzzy reasoning. Because the output fuzzy sets are singleton type, the resulting membership assigned to $\Delta U_m^{J_m}$ in the rule consequent is

$$\mu_{J_m}(\Delta u_m)=\mu_{I_1}(x_1)\cdot\mu_{I_2}(x_2)\cdot\mu_{I_3}(x_3)\cdot\mu_{I_4}(x_4). \quad (2)$$

To obtain crisp $\Delta u_m(k)$ from $\Delta U_m^{J_m}$, the centroid defuzzifier is used. The result is

$$\Delta U_m(k)=a_m\cdot\Delta u_m(k)=a_m\frac{\sum m_{J_m}(\Delta u_m)\cdot J_m V}{m_{J_m}(\Delta u_m)},\qquad m=1,2 \quad (3)$$

where α_m are output scaling factors. New fuzzy controller outputs are computed by adding $\alpha_m\Delta u_m(k)$ to their previous outputs, $U_m(k-1)$:

$$U_m(k)=U_m(k-1)+\Delta U_m(k).$$

2.2 Mamdani Fuzzy PD Controller

A linear PI controller in incremental form has the same structure as a linear PD controller in position form [26]. Thus, with two minor modifications to the configuration of the fuzzy PI controller, one gets the fuzzy PD controller. First, we need to change $x_2(k)$ and $x_4(k)$ respectively to

$$x_2(k)=k_2 d_1(k)=k_2(\Delta e_1(k)-\Delta e_1(k-1)),$$
$$x_4(k)=k_4 d_2(k)=k_4(\Delta e_2(k)-\Delta e_2(k-1)).$$

Second, we need to change $\Delta u_m(k)$ in the rule consequent to $u_m(k)$ (that is, output instead of incremental output). The changes cause the defuzzification result become

$$U_m(k)=a_m\cdot u_m(k)=a_m\frac{\sum m_{J_m}(u_m)\cdot J_m V}{m_{J_m}(u_m)},\qquad m=1,2.$$

3. Results

3.1 Structure Derivation and Analysis of Variable Gains

Using Table 1, and expressions (1) and (2), all 16 membership values for $\Delta U_1^{J_1}$ and $\Delta U_2^{J_2}$ in the 16 fuzzy rules can be obtained. Put them into the defuzzifier (3) and simplify the results. The analytical structure of the fuzzy PI controller can be attained, as stated in the following theorem.

Table 1. Sixteen fuzzy control rules.

IF $x_1(k)=X_1^{I_1}$ AND $x_2(k)=X_2^{I_2}$ AND $x_3(k)=X_3^{I_3}$ AND $x_4(k)=X_4^{I_4}$
THEN $\Delta u_1(k)=\Delta U_1^{J_1}$, $\Delta u_2(k)=\Delta U_2^{J_2}$,
where $J_1=\frac{I_1+I_2}{2}+\delta_1\frac{I_3+I_4}{2}$, $J_2=\delta_2\frac{I_1+I_2}{2}+\frac{I_3+I_4}{2}$

Rule No.	I_1	I_2	I_3	I_4	J_1	J_2
1	1	1	1	1	$1+\delta_1$	$1+\delta_2$
2	1	-1	1	1	δ_1	1
3	-1	1	1	1	δ_1	1
4	-1	-1	1	1	$-1+\delta_1$	$1-\delta_2$
5	1	1	-1	-1	$1-\delta_1$	$-1+\delta_2$
6	1	-1	-1	-1	$-\delta_1$	-1
7	-1	1	-1	-1	$-\delta_1$	-1
8	-1	-1	-1	-1	$-1-\delta_1$	$-1-\delta_2$
9	1	1	1	-1	1	δ_2
10	1	-1	1	-1	0	0
11	-1	1	1	-1	0	0
12	-1	-1	1	-1	-1	$-\delta_2$
13	1	1	-1	1	1	δ_2
14	1	-1	-1	1	0	0
15	-1	1	-1	1	0	0
16	-1	-1	-1	1	-1	$-\delta_2$

Theorem 1. Each incremental output of the TITO Mamdani fuzzy PI controller is the sum of two nonlinear, variable gain PI controllers in incremental form:

$$\Delta U_1(k) = a_1 \cdot \Delta u_1(k) = K_i^1 e_1(k) + K_p^1 \Delta e_1(k) + d_1\left(K_i^2 e_2(k) + K_p^2 \Delta e_2(k)\right)$$
$$\Delta U_2(k) = a_2 \cdot \Delta u_2(k) = d_2\left(K_i^3 e_1(k) + K_p^3 \Delta e_1(k)\right) + K_i^4 e_2(k) + K_p^4 \Delta e_2(k)$$

where the variable proportional-gains (K_p^i) and integral-gains (K_i^i) are

$$K_p^1 = \frac{\alpha_1\beta_1 k_2 H}{2L},\ K_p^2 = \frac{\alpha_1\beta_2 k_4 H}{2L},\ K_p^3 = \frac{\alpha_2\beta_1 k_2 H}{2L},\ K_p^4 = \frac{\alpha_2\beta_2 k_4 H}{2L}$$
$$K_i^1 = \frac{\alpha_1\beta_1 k_1 H}{2L},\ K_i^2 = \frac{\alpha_1\beta_2 k_3 H}{2L},\ K_i^3 = \frac{\alpha_2\beta_1 k_1 H}{2L},\ K_i^4 = \frac{\alpha_2\beta_2 k_3 H}{2L}$$

and

$$b_1 = \frac{16L^8 - L^2(x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)(x_1x_2 + L^2)}{32L^8 - (x_1^2L^2 + x_1^2x_2^2 + x_2^2L^2 + L^4)(x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)} \quad (4)$$

$$b_2 = \frac{16L^8 - L^2(x_1^2L^2 + x_1^2x_2^2 + x_2^2L^2 + L^4)(x_3x_4 + L^2)}{32L^8 - (x_1^2L^2 + x_1^2x_2^2 + x_2^2L^2 + L^4)(x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)}. \quad (5)$$

Based on the relationship between the PI controller in incremental form and the PD controller in position, one can directly obtain the following result:

Theorem 2. Each output of the TITO Mamdani fuzzy PD controller is the sum of two nonlinear, variable gain PD controllers in position form:

$$U_1(k) = a_1 \cdot u_1(k) = K_p^1 e_1(k) + K_d^1 d_1(k) + d_1\left(K_p^2 e_2(k) + K_d^2 d_2(k)\right)$$
$$U_2(k) = a_2 \cdot u_2(k) = d_2\left(K_p^3 e_1(k) + K_d^3 d_1(k)\right) + K_p^4 e_2(k) + K_d^4 d_2(k)$$

where the variable proportional-gains (K_p^i) and derivative-gains (K_d^i) are

$$K_p^1 = \frac{a_1 b_1 k_1 H}{2L}, K_p^2 = \frac{a_1 b_2 k_3 H}{2L}, K_p^3 = \frac{a_2 b_1 k_1 H}{2L}, K_p^4 = \frac{a_2 b_2 k_3 H}{2L}$$
$$K_d^1 = \frac{a_1 b_1 k_2 H}{2L}, K_d^2 = \frac{a_1 b_2 k_4 H}{2L}, K_d^3 = \frac{a_2 b_1 k_2 H}{2L}, K_d^4 = \frac{a_2 b_2 k_4 H}{2L}$$

It should be pointed out that in either case, exchanging $x_1(k)$ with $x_2(k)$ or $x_3(k)$ with $x_4(k)$ does not affect the expressions of β_1 and β_2 and therefore does not affect the expressions of $\Delta U_m(k)$ and $U_m(k)$, too. That is,

$$\beta_p(\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4) = \beta_p(\Delta x_2, \Delta x_1, \Delta x_4, \Delta x_3),$$
p = 1, 2,

where $\beta_p(\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4)$ represents β_p in (4) and (5), respectively. This property can be viewed as a structural duality owing to the sysmetric configuration of the fuzzy controllers with respect to the input variables.

The proportional-gains, integral-gains and derivative-gain vary with β_1 and β_2. The ranges of the gain varivations depend on the ranges of β_1 and β_2. Due to the structual duality, the ranges for β_1 and β_2 are also dual structurally. To determine the range of β_p, the maximal and minimal β_p are needed, which are determined by all $x_i(k)$. Given $x_3(k)$ and $x_4(k)$, the maximal β_1, denoted as $\beta_1^{max}(x_3, x_4)$, occurs when $x_1(k)$ = L and $x_2(k)$ = -L or when $x_1(k)$ = -L and $x_2(k)$ = L:

$$b_1^{max}(x_3, x_4) = \frac{4L^4}{8L^4 - (x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)}.$$

The maximal $\beta_1^{max}(x_3, x_4)$ happens when $|x_3(k)|$ = $|x_4(k)|$ = L, which is

$$b_1^{max}(L, L) = 1.$$

Because of the structural duality,

$$b_1^{max}(L, L) = b_2^{max}(L, L).$$

On the other hand, given $x_3(k)$ and $x_4(k)$, the minimal β_1, denoted as $\beta_1^{min}(x_3, x_4)$, takes place when $x_1(k)$ = $x_2(k)$ = 0:

$$b_1^{min}(x_3, x_4) = \frac{16L^4 - (x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)}{32L^4 - (x_3^2L^2 + x_3^2x_4^2 + x_4^2L^2 + L^4)}$$

with its minimal occuring when $|x_3(k)|$ = $|x_4(k)|$ = L:

$$b_1^{min}(L, L) = \frac{3}{7}.$$

Again, owing to the structural duality,

$$b_1^{min}(L, L) = b_2^{min}(L, L).$$

Therefore, for p = 1, 2,

$$\frac{3}{7} \le \beta_p \le 1.$$

Based on this range, the ranges for K_p^i, K_i^i and K_d^i can be determined accordingly – they are the same as that for b_p.

In our previous publications [21-24], we have derived the anlaytical structures of various SISO Mamdani fuzzy PI/PD controllers and showed that they are nonlinear, variable gain PI/PD controllers. We have also provided in-depth analysis as well as computer simulation to explore why the gain varations, similar to those in this paper, is benefical for control performance enhencement and enables the fuzzy controllers to outperform their linear counterparts. To be concise, we will not repeat the analysis and simulation studies for the TITO fuzzy controllers. Rather, we just briefly summarize the analysis and simulation results. In essense, the variable gains empower the fuzzy controllers to produce stronger control action when the system output is farther away from the setpoint and weaker action when the system output is closer to the setpoint. The combined effect is shorter rise time and settling time with reduced overshoot for nonlinear system control. These benefits and effects are shared by the TITO fuzzy PI/PD controllers in this paper. That is to say that the fuzzy controllers can outpform the TITO linear PI/PD controllers when controlling nonlinear systems.

3.2 Stability Analysis

We now turn our attention to system stability and study local stability of the TITO fuzzy control systems involving the fuzzy PI/PD controllers. Note that $x_i(k) = 0$ for all i is a system equilibrium point, where

$$b_1 = b_2 = \frac{15}{31},$$

and the fuzzy PI/PD controllers becomes linear PI/PD controllers with constant gains. These linear controllers are called the corresponding linear PI/PD controllers of the fuzzy PI/PD controllers. According to Lyapunov's linearization method (e.g., [14]), the fuzzy control system is stable (or unstable) at the equilibrium point if the linearized control system is strictly stable (or unstable). Hence, we have the following result.

Theorem 3. Assume the system under control is continuously differentiable at the equilibrium point. The fuzzy PI (or PD) control system is locally stable (or unstable) if and only if the corresponding linear PI (or PD) control system is strictly stable (or unstable).

It should be emphasized that local stability does not mean system stability at the equilibrium point only. Rather, it means system stability in a region around the equilibrium point. Local stability can be determined without much information and assumption on the fuzzy controller and system. Only two pieces of information are needed: (1) the fuzzy controller structure around the equilibrium point (which is available for the fuzzy controllers in this paper), and (2) the linearizability of the system at the equilibrium point. These simple requirements also make the local stability results for fuzzy control systems practically usable.

Using Theorem 3 offers at least three practically important advantages. First of all, it is a necessary and sufficient condition. Unlike a sufficient or necessary global stability condition, this condition is not conservative and is the tightest possible stability condition. Second, the criterion can be used to determine local stability, not only when the system model is explicitly available, but also when the model is unavailable but is known to be linearizable about the equilibrium point (Most physical systems are linearizable). In the latter case, one can devise a linear PI (or PD) controller and use it to control the nonlinear system. If the resulting control system is observed to be locally stable (unstable), then the same system controlled by a linearizable fuzzy PI (or PD) controller whose gains at the equilibrim point equal the gains of the linear PI (or PD) controller will be locally stable (unstable). This approach is of particular importance as in practice, physical systems are often too complex and/or costly to be precisely modeled. Since any model is merely an approximation to the physical system, it is rational to seek a method capable of determining system stability without the accurate model.

4. Conclusions

We have proved that the analytical structure of the TITO fuzzy PI/PD controllers is the sum of two nonlinear PI/PD controllers whose gains continuously change with system outputs. The characteristics of the gain variation have been analyzed, so have been the local stability of the fuzzy control systems. Based on the methodology developed in this paper, analytical structures of MIMO fuzzy controllers with more inputs and outputs and local stability of the MIMO fuzzy control systems can also be established. However, the task becomes more difficult with the increased number of inputs and outputs.

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Hao Ying is an Associate Professor at Department of Electrical and Computer Engineering, Wayne State University, after he left the faculty of The University of Texas Medical Branch at Galveston. He is also an Advisory Professor of Dong Hua University, Shanghai, China. He obtained the B.S. and M.S. degrees in Electrical Engineering from Dong Hua university in 1982 and 1984, respectively, and the Ph.D. degree in Biomedical Engineering from The University of Alabama at Birmingham in 1990.

Professor Ying has published one research monograph/advanced textbook entitled *Fuzzy Control and Modeling: Analytical Foundations and Applications* (IEEE Press, 2000), and 46 peer-reviewed journal papers. He is a Guest Editor for *Information Sciences*, *International Journal of Intelligent Control and Systems*, and *Acta Automatica Sinaca*. In 1994, he served as a Program Chair for *The International Joint Conference of North American Fuzzy Information Processing Society (NAFIPS) Conference, Industrial Fuzzy Control and Intelligent System Conference, and NASA Joint Technology Workshop on Neural Networks and Fuzzy Logic*. Additionally, he has served as the Publication Chair for the *2000 IEEE International Conference on Fuzzy Systems* and as a Program Committee Member for many other international conferences.