Constructing Nonlinear Variable Gain Controllers via the Takagi–Sugeno Fuzzy Control

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***Abstract*— We investigated analytical structure of the Takagi–Sugeno (TS) type of fuzzy controllers, which was unavailable in the literature. The TS fuzzy controllers we studied employ a new and simplified TS control rule scheme in which all the rule consequent use a common function and are proportional to one another, greatly reducing the number of parameters needed in the rules. Other components of the fuzzy controllers are general: arbitrary input fuzzy sets, any type of fuzzy logic, and the generalized defuzzifer, which contains the popular centroid defuzzifier as a special case. We proved that all these TS fuzzy controllers were nonlinear variable gain controllers and characteristics of the gain variation were parametrized and governed by the rule proportionality. We conducted an in-depth analysis on a class of nonlinear variable gain proportional-derivative (PD) controllers. We present the results to show: 1) how to analyze the characteristics of the variable gains in the context of control; 2) why the nonlinear variable gain PD controllers can outperform their linear counterpart; and 3) how to generate various gain variation characteristics through the manipulation of the rule proportionality.**

***Index Terms*— Fuzzy control, fuzzy controller design, fuzzy systems, nonlinear control, PID control, stability, Takagi–Sugeno fuzzy control, variable gain control.**

I. Introduction

THERE exist two major different types of fuzzy control: the Mamdani type (e.g., [10], [14], [22], [25] and the Takagi-Sugeno (TS) type [18]. They mainly differ in the fuzzy control rule consequent. The Mamdani fuzzy controllers utilize fuzzy sets as the consequent whereas the TS fuzzy controllers employ (linear) functions of input variables as the consequent. Both types of fuzzy control have successfully been applied to solve practical control problems [26]. Presently, almost all the fuzzy controllers are used and treated as black-box controllers that when constructed properly by the trial-and-error method could produce satisfactory results.

Regardless of the type, fuzzy controllers are just conventional nonlinear controllers. Effort has been made to analytically study the Mamdani fuzzy controllers. The topics range from derivation of the fuzzy controllers' structure (e.g., [1], [2], [7], [9], [11], [13], [16], [24], [27]–[31]) to system stability analysis (e.g., [3], [8], [23], [28]) and to system design (e.g., [12], [20], [30]), to just name a few. However, the development of an analytical fuzzy control theory requires more efforts and progress. Especially, in our opinion, an analytical framework relative to conventional control theory is needed in which fuzzy controllers are analyzed and designed using the analysis and design tools in conventional control theory.

Results in analytical aspects of the TS fuzzy controllers are very scarce [19], [21]. There exists no analytical structure of any TS fuzzy controllers in the literature at present, let alone formal relation between the TS fuzzy controllers and conventional controllers such as the widely used proportional-integral-derivative (PID) controllers. Compared to the Mamdani fuzzy controllers, analytical study of the TS fuzzy controllers seems to involve significantly more difficulties. One of the main reasons is that the TS controllers usually have far more adjustable parameters in the rule consequent and the number of the parameters grows exponentially with the increase of the number of input variables. In theory, these parameters provide a means for tuning local control action, resulting in superior control performance. Indeed, to a large extent the power of the TS rule scheme lies in these parameters. However, manual tuning of these parameters in practice could be ineffective, inefficiently, inappropriate, or sometimes even impossible when the parameters are too many. This is the case partially because of the lack of linguistic intuitiveness of the TS rule consequent. Automatic tuning techniques, such as on-line or off-line parameter optimization schemes, may be ineffective and/or inefficient when the dimension of the parameter space is too high. With a large number of parameters, the convergence of the parameter identification may be difficult and system stability could be hard to guarantee.

The objectives of our research presented in this paper were threefold. First, we wanted to make the TS control rule scheme more efficient. We modified the TS rule scheme in such a way that our new scheme, while maintaining the spirit and advantages of the original TS scheme, drastically reduced the number of parameters in the rule consequent. Second, we desired to derive the analytical structure of the TS fuzzy controllers that used our new rule scheme and related the resulting structures to the PID control and variable gain control in conventional control theory. Finally, we wanted to analyze characteristics of the resulting structures in the context of control. The overall goal was to demonstrate that variable gain controllers could be developed via the TS fuzzy control. In other words, the TS fuzzy control provides a viable means to construct various nonlinear controllers with variable gains.

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II. Construction of Variable Gain Controllers via the TS Fuzzy Control

A. Configuration of the TS Fuzzy Controllers in This Study

The TS fuzzy controllers under this investigation use M $(M \geq 1)$ discrete-time input variables, namely $x_1(n), x_2(n), \cdots, x_M(n)$, where n represents sampling time. Variable $x_i(n)$ is fuzzified by P_i $(P_i > 1)$ input fuzzy sets whose membership functions, specified as μ_{ik_i} where $k_i = 1, 2, \cdots, P_i$, are arbitrary. For M input variables, $P_1 \times P_2 \cdots \times P_M$ different combinations of the input fuzzy sets exist and, hence, that many fuzzy rules are needed. We use Ω to represent the total number of the rules and

$$\Omega = P_1 \times P_2 \cdots \times P_M = \prod_{i=1}^{M} P_i.$$

The fuzzy control rules are generated by our newly developed simplified TS fuzzy control rule scheme. In the original TS fuzzy control rule scheme [18], a rule consequent is typically a linear function of input variables and the jth rule looks like

$$\begin{aligned}&\text{IF } x_1(n) \text{ is } A_{1j} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{Mj}\\ &\text{THEN } v_j(n) = a_{0j} + a_{1j}x_1(n) + \cdots + a_{Mj}x_M(n)\end{aligned} \tag{1}$$

where A_{ij}'s are input fuzzy sets, and a_{ij}'s and a_{0j} are adjustable parameters that can be any values. This flexibility in value, however, comes with a severe tradeoff: too many parameters need to be tuned in order to make the fuzzy controllers work properly. More specifically, for Ω rules, each of which has $M+1$ parameters in the rule consequent, there are total

$$\kappa = (M+1)\Omega \tag{2}$$

parameters. For a simple TS fuzzy controller with three input variables (i.e., $M = 3$) and each of them is fuzzified by only two input fuzzy sets (i.e., $P_1 = P_2 = P_3 = 2$, thus, $\Omega = 8$), the total number of the parameters is 32. Even for the simplest useable TS controllers ($M = 2$, $P_1 = P_2 = 2$, and $\Omega = 4$), κ is still as high as 12. Compared to the widely used proportional integral derivative (PID), proportional integral (PI), and proportional derivative (PD) controllers which have only three or two adjustable parameters, the TS controllers using rules in (1) are extremely disadvantageous as far as practicality and ease of use are concerned.

To dramatically reduce the number of the adjustable parameters, we introduce a simplified TS rule scheme as follows:

$$\begin{aligned}R_1&: \text{IF } x_1(n) \text{ is } A_{11} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{M1}\\ &\quad \text{THEN } v_1(n) = k_1(a_0 + a_1x_1(n) + \cdots + a_Mx_M(n))\\ R_2&: \text{IF } x_1(n) \text{ is } A_{12} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{M2}\\ &\quad \text{THEN } v_2(n) = k_2v_1(n)\\ &\qquad\cdots\cdots\\ R_j&: \text{IF } x_1(n) \text{ is } A_{1j} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{Mj}\\ &\quad \text{THEN } v_j(n) = k_jv_1(n)\end{aligned} \tag{3}$$

where R_j represents the jth rule ($1 \leq j \leq \Omega$). Without loss of generality, the value of k_1 is always supposed to be one in this paper. Substituting $v_1(n)$ in R_1 into R_j, the jth rule becomes

$$\begin{aligned}R_j&: \text{IF } x_1(n) \text{ is } A_{1j} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{Mj}\\ &\quad \text{THEN } v_j(n) = k_j(a_0 + a_1x_1(n)\\ &\qquad + \cdots + a_Mx_M(n)).\end{aligned}$$

The proportionality among the rules is fixed and is k_j for the jth rule with respect to the first rule. One sees that all the rule consequent are a common linear function of input variables and they change as the values of input variables vary. In (3), a_0, a_1, $\cdots$, a_M, k_2, k_3, $\cdots$, k_Ω are adjustable design parameters and, hence, the total number of the parameters in the rule consequent is only

$$\gamma = M + \Omega$$

which is much smaller than κ in (2). For instance, when $M = 3$ and $\Omega = 8$, $\gamma = 11$ (but $\kappa = 32$); and when $M = 2$ and $\Omega = 4$, $\gamma = 6$ (but $\kappa = 12$).

We generalize our new rule scheme from linear rule consequent to arbitrary nonlinear rule consequent

$$\begin{aligned}R_1&: \text{IF } x_1(n) \text{ is } A_{11} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{M1}\\ &\quad \text{THEN } v_1(n) = k_1f(x_1(n), \cdots, x_M(n))\\ &\qquad\cdots\cdots\\ R_j&: \text{IF } x_1(n) \text{ is } A_{1j} \text{ AND } \cdots \text{ AND } x_M(n) \text{ is } A_{Mj}\\ &\quad \text{THEN } v_j(n) = k_jv_1(n)\end{aligned} \tag{4}$$

where f can be any nonlinear function. Obviously, the rules described in (3) are only special cases of the rules in (4) when f is the linear function. We call the TS fuzzy control rules in (3) and (4) simplified linear TS fuzzy control rules and simplified nonlinear TS fuzzy control rules, respectively. The simplified control rules, through much simpler than the original TS rules, are very powerful. As will be shown later in this paper, even a few simplified linear rules are able to produce superior nonlinear control to what the linear PID, PI, or PD controllers can possibly offer.

To combine the M membership values of the input fuzzy sets in the rule antecedent in (3) and (4), any type of fuzzy logic AND may be used. We denote such combined membership for consequent $v_j(n)$ in the jth rule as μ_j. The value of μ_j is determined by the membership functions of the input fuzzy sets, the current values of the input variables and the type of fuzzy logic AND used.

We use the generalized defuzzifier [6] to combine $v_j(n)$ for $j = 1, 2, \cdots, \Omega$. After the defuzzification, the output of the fuzzy controllers is

$$u(n) = \frac{\sum_{j=1}^{\Omega}(\mu_j)^\alpha \cdot v_j(n)}{\sum_{j=1}^{\Omega}(\mu_j)^\alpha}. \tag{5}$$

Different defuzzification results can be obtained by using different α value [6], where $0 \leq \alpha < +\infty$. The popular centroid defuzzifier and mean of maximum defuzzifier are just two special cases when $\alpha = 1$ and ∞, respectively. Equation (5) can be written as

$$\begin{aligned}u(n) &= \frac{\sum_{j=1}^{\Omega}(\mu_j)^\alpha \cdot v_j(n)}{\sum_{j=1}^{\Omega}(\mu_j)^\alpha} = \frac{\sum_{j=1}^{\Omega}(\mu_j)^\alpha \cdot k_j}{\sum_{j=1}^{\Omega}(\mu_j)^\alpha}\\ &\quad \times f(x_1, \cdots, x_M) = G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot f(\vec{x})\end{aligned} \tag{6}$$

where the meanings of the vectors are

$$\vec{k} = (k_1, k_2, \cdots, k_\Omega),$$
$$\vec{\mu} = (\mu_{11}, \cdots, \mu_{1P_1}, \cdots, \mu_{M1}, \cdots, \mu_{MP_M}),$$
$$\vec{x} = (x_1(n), x_2(n), \cdots, x_M(n))$$

and

$$G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) = \frac{\sum_{j=1}^{\Omega} (\mu_j)^\alpha \cdot k_j}{\sum_{j=1}^{\Omega} (\mu_j)^\alpha}.$$

Here, the purpose of using $\vec{x}$, $\vec{k}$, and $\vec{\mu}$ is to concisely represent the many variables of the nonlinear function G.

We should point out that if $v_j(n)$ in (3) or (4) represents output of the fuzzy controllers, then $u(n)$ in (5) is the output of the fuzzy controllers at time n. On the other hand, if $v_j(n)$ represents change in the output of the fuzzy controllers, then $u(n)$ in (5) is the change in the output of the fuzzy controllers at time n. To distinguish them, we denote the output and change in output as $u(n)$ and $\Delta u(n)$, respectively, in the rest of this paper. Note that $u(n) = u(n-1) + \Delta u(n)$.

B. The TS Fuzzy Controllers Are Nonlinear Variable Gain Controllers

The following result links the structure of the TS fuzzy controllers configured in Section II-A to that of variable gain controllers in conventional control theory.

Theorem The TS fuzzy controllers using the simplified TS control rules (linear or nonlinear) are nonlinear variable gain controllers.

Proof: According to (6), the TS fuzzy controllers with the simplified TS control rules are described by

$$u(n) = G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot f(\vec{x}).$$

Note that α, $\vec{k}$, and $\vec{\mu}$ are fixed once a specific fuzzy controller is constructed. Hence, the fuzzy controllers are nonlinear controllers (i.e., $f(\vec{x})$) whose gains vary due to the change of $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$ with respect to the input variables $\vec{x}$. ■

This result is important as it provides a theoretical ground for using qualitative information in the forms of fuzzy control rules, membership functions, and fuzzy logic to develop a variety of variable gain controllers with desired gain variation characteristics. The gain variation is caused by the variation of $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$ whose value changes with the values of $\vec{x}$. Characteristics of the gain variation are governed by the simplified TS rules (i.e., $\vec{k}$), the membership functions of the input fuzzy sets (i.e., $\vec{\mu}$) and the generalized defuzzifier (i.e., α). Different $\vec{k}$, $\vec{\mu}$, and α result in different gain variation characteristics.

Although from theory standpoint, nonlinear rule consequent are more general and, thus, more powerful, various practical applications with the original TS fuzzy rules have shown that linear rule consequent are much easier to use and the resulting fuzzy controllers can satisfactorily solve rather complex nonlinear control problems. Therefore, in the rest of this paper, we will focus on the fuzzy controllers with the simplified linear TS fuzzy rules and their relationship with the PID controller, especially the PD controller.

C. The Relationship Between the TS Fuzzy Controllers with the Simplified Linear TS Fuzzy Rules and the PID Controller

When the simplified linear TS rules in (3) are employed and when we use $\Delta u(n)$ instead of $u(n)$ in (6)

$$\Delta u(n) = G(\alpha, \vec{k}, \vec{\mu}, \vec{x})(a_0 + a_1 x_1 + \cdots + a_M x_M) \quad (7)$$

which means that the fuzzy controllers become nonlinear controllers with variable gain $G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot a_i$ for $x_i(n)$, where $i = 1, 2, \cdots, M$.

In (7), $a_0 + a_1 x_1 + \cdots + a_M x_M$ represents linear controllers which contain the linear PID controller as a special case (we assume here $a_0 = 0$). The following three-input variables are used by the discrete-time PID control in incremental form

$$x_1(n) = \mathrm{SP}(n) - y(n)$$
$$x_2(n) = x_1(n) - x_1(n-1)$$
$$x_3(n) = x_2(n) - x_2(n-1)$$

where $\mathrm{SP}(n)$ is a setpoint/reference signal for process output and $y(n)$ is the process output at sampling time n. Using these three input variables and letting $a_0 = 0$ in (7), we obtain

$$\Delta u(n) = G(\alpha, \vec{k}, \vec{\mu}, \vec{x})(a_1 x_1(n) + a_2 x_2(n) + a_3 x_3(n)). \quad (8)$$

Note that the linear discrete-time PID controller in incremental form is (e.g., [15])

$$\Delta u_{\mathrm{PID}}(n) = \bar{K}_p x_2(n) + \bar{K}_i x_1(n) + \bar{K}_d x_3(n) \quad (9)$$

where $\bar{K}_p$, $\bar{K}_i$, and $\bar{K}_d$ are constant gains named proportional-gain, integral-gain, and derivative-gain, respectively. Comparing (8) with (9), one sees that the fuzzy controllers in (8) become nonlinear PID controllers with variable proportional-gain, integral-gain, and derivative-gain being $G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot a_2$, $G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot a_1$ and $G(\alpha, \vec{k}, \vec{\mu}, \vec{x}) \cdot a_3$, respectively.

The TS fuzzy controllers with the simplified linear TS rules become linear controllers if in (3) k_j is a nonzero constant B for $j = 1, 2, \cdots, \Omega$. This is because when $k_j = \mathrm{B}$ for all the values of j, the rule consequent of all Ω rules become the same and is $\mathrm{B}(a_1 x_1(n) + \cdots + a_M x_M(n))$. Note that the antecedents of the rules list all the possible input states. That means when $k_j = \mathrm{B}, u(n) = \mathrm{B}(a_1 x_1(n) + \cdots + a_M x_M(n))$, regardless of value of the input variables, meaning that the fuzzy controllers become linear controllers. Practically speaking, though, no one would want to use fuzzy control to realize linear control. The significance and real value of the TS fuzzy controllers presented in this paper lie in the gain variation, which, in turn, depends on $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$.

Explicit formulation of $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$ for a given fuzzy controller may analytically be derived. The derivability depends on the complicity of the simplified TS rules, the input fuzzy sets and the defuzzifier that are used. Once $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$ is analytically available, its characteristics can be analyzed in the context of control.

In the next section, we will conduct an in-depth analysis on a class of nonlinear variable gain PD controllers to demonstrate the usefulness of the gain variation. There were two main reasons why we chose the nonlinear PD controllers. First, the PD control is one of the most popular and useful control schemes in industries. In fact, the PID, PI, and PD controllers are currently controlling some 90% of industrial processes [5]. Any new controller that is comparably simple in the gains tuning and has potential to outperform it, especially when nonlinear processes are involved, would be of great practical value. Second, gain variation with two input variables can be presented and visualized graphically along with analytical analysis. The results obtained may be extended to TS fuzzy controllers with three-input variables or more, but graphical presentation and visualization of the gain variation with all the input variables is impossible.

III. Analysis of a Class of Nonlinear Variable Gain PD Controllers Constructed via the Fuzzy Controllers That Use the Simplified Linear TS Rules

In this section, we show: 1) how to analyze the characteristics of the variable gains in the context of control; 2) why the variable gain PD controllers can outperform their linear counterpart; and 3) how to generate various gain variation characteristics through the manipulation of the rule proportionality.

The linear discrete-time PD controller in position form is

$$u_{\mathrm{PD}}(n) = \bar{K}_p x_1(n) + \bar{K}_d x_2(n) \tag{10}$$

which is similar to the linear PI controller in incremental form, which can be obtained by letting $\bar{K}_d = 0$ in (9)

$$\Delta u_{\mathrm{PI}}(n) = \bar{K}_p x_2(n) + \bar{K}_i x_1(n).$$

One sees that the PD controller in position form becomes the PI controller in incremental form if $x_1(n)$ and $x_2(n)$ exchange their positions and $\bar{K}_d$ is replaced by $\bar{K}_i$. Hence, the results developed below can easily be extended to cover the corresponding class of nonlinear variable gain PI controllers.

A. Configuration of the TS Fuzzy Controllers

Each of the two input variables $x_1(n)$ and $x_2(n)$ is fuzzified by two-input fuzzy sets named "positive" and "pegative," respectively. The mathematical definitions of the "positive" and "negative" fuzzy sets are identical for the input variables and are (Fig. 1)

$$\mu_P(x_i) = \begin{cases} 0, & x_i < -L \\ \frac{x_i + L}{2L}, & -1 \le x_i \le L \\ 1, & x_i > L \end{cases}$$

and

$$\mu_N(x_i) = \begin{cases} 1, & x_i < -L \\ \frac{-x_i + L}{2L}, & -L \le x_i \le L \\ 0, & x_i > L \end{cases} \tag{11}$$

where $i = 1$ or 2 and the subscripts P and N represent "positive" and "negative," respectively. The value of L affects the control performance and should be carefully chosen by the controller designer.

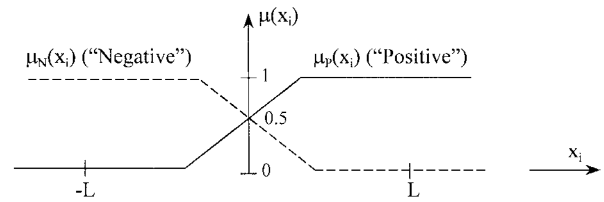

Fig. 1. The graphical definition of the input fuzzy sets used in this paper. The input variables, specified as x_i where $i = 1$ or 2, are fuzzified by the identical input fuzzy sets. The corresponding mathematical definitions are given in (11).

The fuzzy controllers use following four simplified linear TS control rules:

R_1: IF $x_1(n)$ is Positive AND $x_2(n)$ is Positive THEN $v_1(n) = k_1(a_1 x_1(n) + a_2 x_2(n))$
R_2: IF $x_1(n)$ is Positive AND $x_2(n)$ is Negative THEN $v_2(n) = k_2 v_1(n)$
R_3: IF $x_1(n)$ is Negative AND $x_2(n)$ is Positive THEN $v_3(n) = k_3 v_1(n)$
R_4: IF $x_1(n)$ is Negative AND $x_2(n)$ is Negative THEN $v_4(n) = k_4 v_1(n)$ (12)

where $k_1 = 1$, as we stated earlier. In the rules, $v_j(n)$ $(j = 1, 2, 3, 4)$ represents output of the controllers. We use Zadeh fuzzy logic AND to evaluate the AND in the rules, and use the centroid defuzzifier to produce the output of the fuzzy controllers

$$u(n) = \frac{\sum_{j=1}^{4} \mu_j \cdot v_j(n)}{\sum_{j=1}^{4} \mu_j}. \tag{13}$$

B. Derivation of the Analytical Structure of the Fuzzy Controllers

Substituting $v_j(n)$ into (13), we obtain

$$\begin{aligned} u(n) &= \frac{\sum_{j=1}^{4} \mu_j \cdot v_j(n)}{\sum_{j=1}^{4} \mu_j} = v_1(n) \frac{\sum_{j=1}^{4} \mu_j \cdot k_j}{\sum_{j=1}^{4} \mu_j} \\ &= F(x_1, x_2)(a_1 x_1(n) + a_2 x_2(n)) \end{aligned} \tag{14}$$

where

$$F(x_1, x_2) = \frac{\sum_{j=1}^{4} \mu_j \cdot k_j}{\sum_{j=1}^{4} \mu_j}. \tag{15}$$

Here, we use $F(x_1, x_2)$ instead of $G(\alpha, \vec{k}, \vec{\mu}, \vec{x})$ to emphasize that once the fuzzy controllers are constructed, the gain variation only depends on x_1 and x_2. Comparing (14) with (10), one sees that the fuzzy controllers are nonlinear PD controllers with

$$K_p(x_1, x_2) = a_1 \cdot F(x_1, x_2)$$

and

$$K_d(x_1, x_2) = a_2 \cdot F(x_1, x_2) \tag{16}$$

being the variable proportional gain and derivative gain, respectively. Apparently, the variable gains are essentially determined by the nonlinear function $F(x_1, x_2)$. Thus, to analyze characteristics of the nonlinear variable gain PD

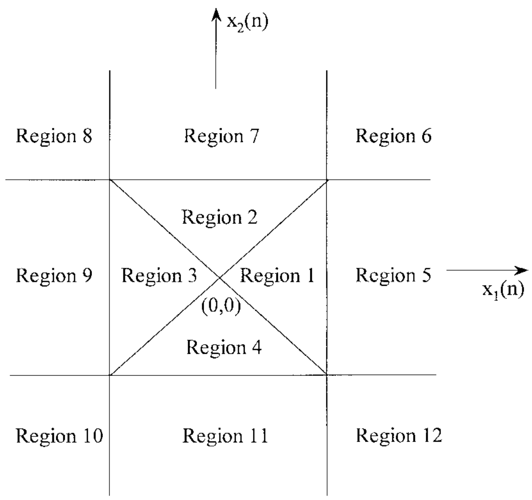

Fig. 2. The input space is divided into 12 different regions. In each region, an explicit expression of $F(x_1, x_2)$ can be derived. The resulting expressions are shown in Table I.

controllers, we first need to derive the analytical expression of $F(x_1, x_2)$.

Since k_1 to k_4 in (15) are known, the key for the derivation is to determine μ_j where $j = 1, 2, 3, 4$. We divide the $x_1(n)$-$x_2(n)$ plane into 12 regions and label them from Region 1 to Region 12, as shown in Fig. 2. The purpose of dividing the input space into the 12 regions is to achieve, in each region, an unique inequality relationship between the two memberships being ANDed in each rule. The AND in each of the four rules is evaluated using Zadeh fuzzy logic AND, which yields the smaller membership among the two memberships being ANDed as μ_j [27]. For example, in Region 1, $\mu_1 = \mu_P(x_2)$, $\mu_2 = \mu_N(x_2)$, $\mu_3 = \mu_N(x_1)$, and $\mu_4 = \mu_N(x_1)$. Having obtained μ_j $(j = 1, 2, 3, 4)$ for all the 12 regions, we substitute the definitions of $\mu_p(x_1)$, $\mu_N(x_1)$, $\mu_p(x_2)$, and $\mu_N(x_2)$ given in (11) into (15) simplify the resulting expressions. We obtain the explicit expression of $F(x_1, x_2)$ for the 12 regions, as given in Table I.

According to Table I, $F(x_1, x_2)$ for Regions 1–4 varies with both $x_1(n)$ and $x_2(n)$. In Regions 7 and 11, $F(x_1, x_2)$ varies only with $x_1(n)$, while in Regions 5 and 9, $F(x_1, x_2)$ changes only with $x_2(n)$. Hence, in Regions 1–5, 7, 9, and 11, the fuzzy controllers are nonlinear variable gain PD controllers. $F(x_1, x_2)$ is a constant k_1, k_1k_3, k_1k_4, and k_1k_2 in Regions 6, 8, 10, and 12, respectively, making the fuzzy controllers linear PD controllers.

C. Characteristics of the Variable Gains of the Nonlinear PD Controllers

Characteristics of the variable gains are determined by the characteristics of $F(x_1, x_2)$, which, in turn, are determined by the values of k_1, k_2, k_3, k_4, and L. As a result, different nonlinear PD controllers can be generated. $F(x_1, x_2)$ in Regions 1–4 are most nonlinear and, hence, are most important. Stable fuzzy control systems should operate in these regions most of time because $(x_1, x_2) = (0, 0)$ is the system equilibrium point. These four regions are bounded by the square $[-L, L] \times [-L, L]$. The value of L affects the overall control performance. A smaller value of L makes the system to stay outside these four regions more often, whereas a larger value does the opposite. To achieve satisfactory control performance, the value of L should appropriately be selected.

TABLE I
THE EXPLICIT EXPRESSION OF $F(x_1, x_2)$ DERIVED FOR THE 12 DIFFERENT REGIONS THAT DIVIDE UP THE $x_1(n)$-$x_2(n)$ PLANE, SHOWN IN FIG. 2

Region No.	$F(x_1,x_2)$
1	$\frac{A[B - (k_3 + k_4)x_1 + (1 - k_2)x_2]}{2L - x_1}$
2	$\frac{A[B + (1 - k_3)x_1 - (k_2 + k_4)x_2]}{2L - x_2}$
3	$\frac{A[B + (1 + k_2)x_1 + (k_3 - k_4)x_2]}{2L + x_1}$
4	$\frac{A[B + (k_2 - k_4)x_1 + (1 + k_3)x_2]}{2L + x_2}$
5	$A[(1 + k_2)L + (1 - k_2)x_2]$
6	k_1
7	$A[(1 + k_3)L + (1 - k_3)x_1]$
8	k_1k_3
9	$A[(k_3 + k_4)L + (k_3 - k_4)x_2]$
10	k_1k_4
11	$A[(k_2 + k_4)L + (k_2 - k_4)x_1]$
12	k_1k_2

Note: $A = \frac{k_1}{2}$ and $B = (1 + k_2 + k_3 + k_4)L$

According to Table I, the characteristics of $F(x_1, x_2)$ are also parametrized and governed by k_1, k_2, k_3, and k_4. Consequently, whether the gain variation is sensible in the context of control depends on the values of k_j. We say that the simplified linear TS control rules in (12) symmetrical if $R_1 = R_4$ and $R_2 = R_3$ or, equivalently, if $k_1 = k_4 = 1$ and $k_2 = k_3$. Note that the symmetry is in terms of input state. $F(x_1, x_2)$ corresponding to the symmetrical control rules is shown in Table II, which reveals that $F(x_1, x_2)$ is symmetrical about the lines $x_1(n) = x_2(n)$ and $x_1(n) = -x_2(n)$. This is the case because $F(x_1, x_2) = F(x_2, x_1)$ and $F(x_1, x_2) = F(-x_2, -x_1)$. These two symmetrical characteristics mean that the proportional gain and derivative gain also change symmetrically in terms of these two lines.

From Table II, one can easily find out that $F(0, 0) = (1 + k_2 + k_3 + k_4)/4$, $F(L, L) = 1, F(-L, L) = k_3, F(-L, -L) = k_4$, and $F(L, -L) = k_2$. Furthermore, $F(x_1, x_2)$ is a continuous function and the continuity holds on the boundaries of any adjacent regions of the 12 regions.

D. Analysis of the Characteristics of the Variable Gains When k_j Are of Specific Values—Some Examples

We now analyze the characteristics of $F(x_1, x_2)$ when k_j are of some specific values. The objectives were: 1) to use these k_j values as examples to exhibit useful characteristics of $F(x_1, x_2)$ in the context of control and 2) to demonstrate

TABLE II
THE EXPLICIT EXPRESSION OF $F(x_1, x_2)$ WHEN THE SIMPLIFIED LINEAR TS FUZZY RULES IN (12) ARE SYMMETRICAL. THE RESULTS ARE OBTAINED FROM TABLE I BY LETTING $k_1 = k_4 = 1$ AND $k_3 = k_2$

Region No.	$F(x_1,x_2)$
1	$\frac{A[B - (1+k_2)x_1 + (1-k_2)x_2]}{2L - x_1}$
2	$\frac{A[B + (1-k_2)x_1 - (1+k_2)x_2]}{2L - x_2}$
3	$\frac{A[B + (1+k_2)x_1 - (1-k_2)x_2]}{2L + x_1}$
4	$\frac{A[B - (1-k_2)x_1 + (1+k_2)x_2]}{2L + x_2}$
5	$A[(1+k_2)L + (1-k_2)x_2]$
6	1
7	$A[(1+k_2)L + (1-k_2)x_1]$
8	k_2
9	$A[(1+k_2)L - (1-k_2)x_2]$
10	1
11	$A[(1+k_2)L - (1-k_2)x_1]$
12	k_2

Note: $A = \frac{1}{2}$ and $B = 2(1+k_2)L$

TABLE III
THE RESULTING EXPLICIT EXPRESSION OF $F(x_1, x_2)$ WHEN $k_2 = 0$ IN TABLE II

Region No.	$F(x_1,x_2)$
1	$\frac{2L - x_1 + x_2}{2(2L - x_1)}$
2	$\frac{2L + x_1 - x_2}{2(2L - x_2)}$
3	$\frac{2L + x_1 - x_2}{2(2L + x_1)}$
4	$\frac{2L - x_1 + x_2}{2(2L + x_2)}$
5	$\frac{L + x_2}{2}$
6	1
7	$\frac{L + x_1}{2}$
8	0
9	$\frac{L - x_2}{2}$
10	1
11	$\frac{L - x_1}{2}$
12	0

that the gain variation empowers the nonlinear PD controllers to outperform their linear counterpart.

Our first and main example is the characteristics of the gain variation when $k_1 = k_4 = 1$ and $k_2 = k_3 = 0$. Under these values, four symmetrical rules are formed

R_1: IF $x_1(n)$ is Positive AND $x_2(n)$ is Positive
THEN $v_1(n) = a_1 x_1(n) + a_2 x_2(n)$
R_2: IF $x_1(n)$ is Positive AND $x_2(n)$ is Negative
THEN $v_2(n) = 0$
R_3: IF $x_1(n)$ is Negative AND $x_2(n)$ is Positive
THEN $v_3(n) = 0$
R_4: IF $x_1(n)$ is Negative AND $x_2(n)$ is Negative
THEN $v_4(n) = a_1 x_1(n) + a_2 x_2(n)$.

Substituting $k_2 = 0$ into Table II, the resulting $F(x_1, x_2)$ is given in Table III. Without loss of generality, we assume $L = 1$ and plot $F(x_1, x_2)$ in Fig. 3 with respect to $x_1(n)$ and $x_2(n)$ whose ranges are $[-2L, 2L]$. The symmetries and continuity of $F(x_1, x_2)$ are clearly seen. Also, one notices that: 1) $F(0,0) = 0.5$; 2) $F(x_1, x_2)$ reaches its maximum one at (L, L) and $(-L, -L)$; and 3) $F(x_1, x_2)$ achieves its minimum zero at $(L, -L)$ and $(-L, L)$. The symmetries make analysis of $F(x_1, x_2)$ simpler as one only needs to study $F(x_1, x_2)$ in Regions 1, 5, 6, and 12 and the analysis results can directly be applied to the remaining regions.

In Regions 1–5, 7, 9, and 11, the fuzzy controllers are nonlinear variable gain PD controllers, and in Regions 6, 8, 10, or 12 the fuzzy controllers are linear PD controllers. We define the proportional gain and derivative gain [described in (16)] at $(0, 0)$ as steady-state proportional gain, denoted as $K_p(0, 0)$, and derivative gain, designated as $K_d(0, 0)$, respectively. When the input variables are of any other values, the gains are defined as dynamic ones. We now study the gain variation in the context of control and in comparison with the gains of the corresponding linear PD controller.

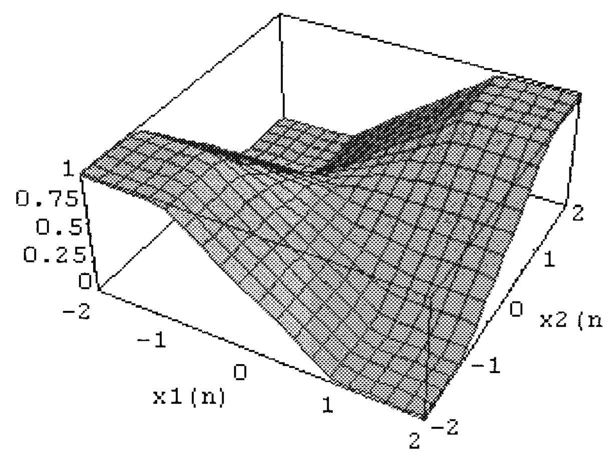

Fig. 3. A three-dimensional (3-D) plot of $F(x_1, x_2)$ whose expressions corresponding to the 12 different regions in the $x_1(n)$-$x_2(n)$ plane are given in Table III. Without loss of generality, L is assumed to be one and $x_1(n)$ and $x_2(n) \in [-2L, 2L]$. $F(x_1, x_2)$ is symmetrical about the lines $x_1(n) = x_2(n)$ and $x_1(n) = -x_2(n)$. $F(x_1, x_2)$ is most nonlinear and of the most importance when $x_1(n)$ and $x_2(n)$ are in the square area of $[-L, L] \times [-L, L]$.

To make the comparison as fair as possible, we make the steady-state gains of the nonlinear PD controllers equal to those of their linear counterpart, defined in (10). That is, we set $a_1 = 2\bar{K}_p$ and $a_2 = 2\bar{K}_d$ so that $K_p(0, 0) = \bar{K}_p$ and $K_d(0, 0) = \bar{K}_d$. When both $x_1(n)$ and $x_2(n)$ are in the first or third quadrant of the $x_1(n)$-$x_2(n)$ plane [but not at (0, 0)],

$$K_p(x_1, x_2) > K_p(0, 0) = \bar{K}_p$$

and

$$K_d(x_1, x_2) > K_d(0, 0) = \bar{K}_d.$$

Because of the symmetries of $F(x_1, x_2)$, we only need to analyze it in the first quadrant in which the process output is below the setpoint and is moving farther away from the setpoint. From control standpoint, an increment in controller output is desirable. The farther the process output is below the setpoint and/or the faster the process output is leaving the setpoint, the greater the increment should be. On the other hand, when the process output is near (0, 0), a smaller increment in control action is advantageous to avoid possible instability due to excessively large controller output. The gain variation of the fuzzy controllers obviously achieve these control strategies in a continuous and smooth fashion.

When $x_1(n)$ and $x_2(n)$ are in the second or fourth quadrant of the $x_1(n)$-$x_2(n)$ plane (excluding the origin),

$$K_p(x_1, x_2) < K_p(0, 0) = \bar{K}_p$$

and

$$K_d(x_1, x_2) < K_d(0, 0) = \bar{K}_d.$$

Again, because of the symmetries of $F(x_1, x_2)$, we only have to study $F(x_1, x_2)$ in the fourth quadrant in which the process output is below the setpoint but is moving toward the setpoint. According to Fig. 3, the controller output is reduced due to the gain variation, which is desirable from standpoint of avoiding possible excessive large control action that could create unwanted oscillation of the process output around the setpoint. Loosely speaking, the farther $(x_1(n), x_2(n))$ is away from (0, 0), the smaller $K_p(x_1, x_2)$ and $K_d(x_1, x_2)$ are compared with $K_p(0, 0)$ and $K_d(0, 0)$, respectively. The gains become zero if the process output is approaching the setpoint too rapidly (i.e., $x_2(n) \leq -L$), resulting in zero controller output.

The above insightful analysis indicates that the gain variation is desirable in the context of control and it empowers the fuzzy controllers to produce superior performance to that of the linear PD controller. This is especially the case when controlling nonlinear processes or processes with time delay. For details in this regard, the reader is referred to [27] and [28] where theoretical analysis as well as computer simulation were conducted for the nonlinear variable gain PI controllers obtained via the Mamdani fuzzy control.

Generally speaking, desired characteristics of the gain variation may be obtained through proper selection of k_j values. During the selection, the controller designer needs to keep in mind that $F(0, 0) = (1 + k_2 + k_3 + k_4)/4$, $F(L, L) = k_1$, $F(-L, L) = k_3$, $F(-L, -L) = k_4$, and $F(L, -L) = k_2$. These five $F(x_1, x_2)$ values determine some major aspects of $F(x_1, x_2)$. The general guideline for selecting k_j values is that they should be such that $F(0, 0)$, $F(L, L)$, $F(-L, L)$, $F(-L, -L)$, and $F(L, -L)$ fit well what the designer wants at these five particular points of the input space.

For instance, we can make use of the gain variation to realize a variety of biased nonlinear PD control strategies in a continuous and smooth fashion. Fig. 4 shows such an example where $k_1 = 1$, $k_2 = k_3 = 0$, and $k_4 = 1/3$. $F(x_1, x_2)$ is symmetrical about the line $x_1(n) = x_2(n)$, but is asymmetrical about the line $x_1(n) = -x_2(n)$. $F(-L, -L) = 1/3$ and $F(0, 0) = 1/3$, which indicate that: 1) the controller gains at (L, L) are three times as large as those at (0, 0) or $(-L, -L)$

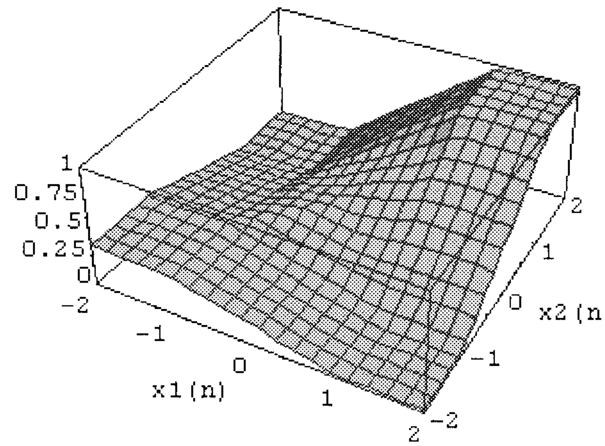

Fig. 4 A 3-D plot of $F(x_1, x_2)$ when $k_1 = 1, k_2 = k_3 = 0$, and $k_4 = 1/3$. L is assumed to be one and $x_1(n)$ and $x_2(n) \in [-2L, 2L]$. $F(x_1, x_2)$ is symmetrical about the line $x_1(n) = x_2(n)$ but is asymmetrical about the line $x_1(n) = -x_2(n)$, producing a desired biased gain variation. $F(x_1, x_2)$ at (L, L) are three times as large as those at (0, 0) or $(-L, -L)$, indicating the gains of this nonlinear PD control can be up to three times larger in the first quadrant than at (0, 0) or $(-L, -L)$. $F(x_1, x_2)$ in the third quadrant is quite "flat" (note $F(0, 0) = F(-L, -L)$), indicating the gain variation is insignificant.

and 2) $F(x_1, x_2)$ in the third quadrant is quite "flat" (because $F(0, 0) = F(-L, -L)$), meaning the gain variation in that quadrant is very small. According to Fig. 4, $F(x_1, x_2)$ in the first quadrant is much bigger and more steep than that in the third quadrant. Using this biased gain variation, undershoot of the process output should be (much) less than the overshoot. This is because the control action is much stronger when $x_1(n)$ and $x_2(n)$ are in the first quadrant than when they are in the third quadrant.

Using different values for k_2 and k_3, one can achieve biased control strategies for the second and fourth quadrants of the $x_1(n)$-$x_2(n)$ plane. Fig. 5 demonstrates such an example where $k_1 = 1$, $k_2 = 1/8$, $k_3 = 1/2$, and $k_4 = 1$. One sees that the gains in the second quadrant are significantly larger than those in the fourth quadrant. $F(x_1, x_2)$ is symmetrical about the line $x_1(n) = -x_2(n)$. As the last example, we demonstrate (in Fig. 6) an $F(x_1, x_2)$ that is asymmetrical with respect to both the lines $x_1(n) = x_2(n)$ and $x_1(n) = -x_2(n)$. In this example, $k_1 = 1$, $k_2 = 1/8$, $k_3 = 1/4$, and $k_4 = 1/2$ (i.e., $k_1 \neq k_4$ and $k_2 \neq k_3$). The analytical expressions of $F(x_1, x_2)$ in these examples can directly be obtained from Table I using the k_j values and the meaning of $F(x_1, x_2)$ can easily be interpreted in the context of control, just as what we did in the first example ($k_1 = k_4 = 1$, $k_2 = k_3 = 0$).

Finally, we point out that compared with the nonlinear variable gain PI and PD controllers derived from some Mamdani fuzzy controllers [4], [12], [27], [28], the nonlinear PD controllers (as well as the corresponding nonlinear PI controllers, as we pointed out in the early portion of Section III) in this paper are much more diverse in gain variation characteristics. This is directly owing to the use of our simplified TS rules that parametrize the characteristics of the gain variation. As a result, there are an infinitely large number of different gain variation characteristics. In sharp contrast, the nonlinear PI controllers in [27], [28], and the nonlinear PD controllers in

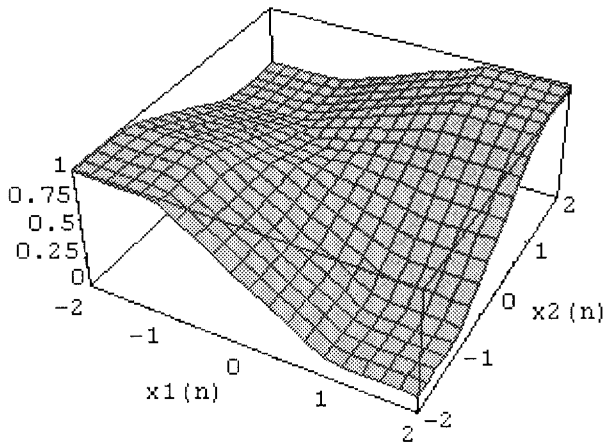

Fig. 5. A 3-D plot of $F(x_1, x_2)$ when $k_1 = 1$, $k_2 = 1/8$, $k_3 = 1/2$, and $k_4 = 1$. L is assumed to be one and $x_1(n)$ and $x_2(n) \in [-2L, 2L]$. $F(x_1, x_2)$ is symmetrical about the line $x_1(n) = -x_2(n)$, but is asymmetrical about the line $x_1(n) = x_2(n)$. The gains in the second quadrant are significantly larger than those in the fourth quadrant.

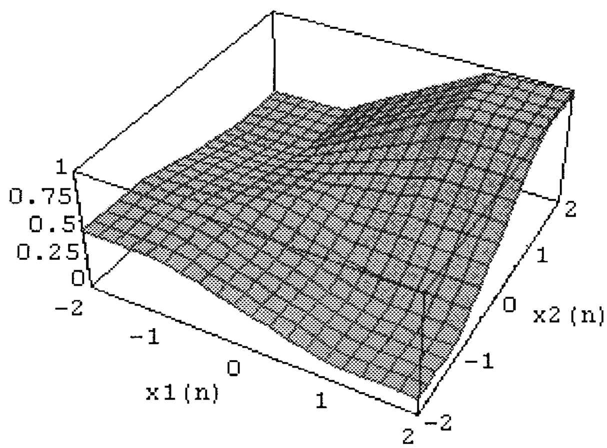

Fig. 6. A 3-D plot of $F(x_1, x_2)$ when $k_1 = 1$, $k_2 = 1/8$, $k_3 = 1/4$, and $k_4 = 1/2$. L is assumed to be one and $x_1(n)$ and $x_2(n) \in [-2L, 2L]$. $F(x_1, x_2)$ is asymmetrical with respect to both the lines $x_1(n) = x_2(n)$ and $x_1(n) = -x_2(n)$. This is because $k_1 \neq k_4$ and $k_2 \neq k_3$.

[12] can generate only a few. As a result, the nonlinear PD/PI controllers constructed via the TS fuzzy controllers are capable of offering more and better solutions to a wider variety of (nonlinear) control problems.

The local stability of the fuzzy control systems involving the nonlinear variable gain PD/PI controllers can be analyzed using the Lyapunov lineralization method [17] (see [28] for details). This is because the nonlinear PD controllers, once linearized around the equilibrium point, become the linear PD controller. The global stability can also be studied using various classical analysis tools such as the small gain theorem, as we have already done [3]. More detailed stability analysis is not included in this paper as our main intention was to develop the relationship between the TS fuzzy controllers and variable gain controllers.

IV. Conclusion

In this paper, we have theoretically proved that the fuzzy controllers using our newly introduced simplified TS rules are nonlinear variable gain controllers. The TS fuzzy controllers are quite general in that they use arbitrary input fuzzy sets, any type of fuzzy logic, and the generalized defuzzifer. The characteristics of the gain variation are parametrized and governed by the rule proportionality. These results lays a solid basis for using experts control knowledge and experience in the forms of fuzzy control rules, membership functions, and fuzzy logic to construct a variety of nonlinear variable gain controllers with desired gain variation characteristics.

Our simplified TS fuzzy rule scheme, while persevering the spirit and advantages of the original TS fuzzy rule scheme, greatly reduces the number of adjustable parameters in the rule consequent. The new rule scheme is simpler, more efficient, and, yet, still powerful. Employing the simplified linear TS rules the fuzzy PD controllers in this paper and the fuzzy PI and PID controllers investigated in our other papers [32], [33] can outperform their linear counterparts. We have also briefly discussed the local and global stability of the fuzzy control systems in this paper.

The results presented in this paper have greatly been generalized in [34]. We have investigated, in relation to some popular controllers, models and filters, analytical structure of a general class of multi-input single-output TS fuzzy systems that use the simplified linear TS rules. We have constructively proved that the general TS fuzzy systems are universal approximators.

The results presented in this paper, being the first ones, bridge the wide knowledge gap, existing in the current literature regarding analytical structure of the TS fuzzy controllers and their possible connection with conventional controllers, including the PID controller.

References

[1] F. Bouslama and A. Ichikawa, “Fuzzy control rules and their natural control laws,” *Fuzzy Sets Syst.*, vol. 48, pp. 65–86, 1992.
[2] J. J. Buckley, “Theory of the fuzzy controller: An introduction,” *Fuzzy Sets Syst.*, vol. 51, pp. 249–258, 1992.
[3] G. Chen and H. Ying, “BIBO stability of nonlinear fuzzy PI control systems,” *J. Intell. Fuzzy Syst.*, to be published.
[4] G. Chen, “Conventional and fuzzy PID controllers: An overview,” *Int. J. Intell. Contr. Syst.*, vol. 1, no. 2, pp. 235–246, 1996.
[5] P. B. Deshpande, “Improve quality control on-line with PID controllers,” *Chem. Eng. Progress*, pp. 71–76, May 1991.
[6] D. P. Filev and R. R. Yager, “A generalized defuzzification method via BAD distributions,” *Int. J. Intell. Syst.*, vol. 6, pp. 687–697, 1991.
[7] A. E. Hajjaji and A. Rachid, “Explicit formulas for fuzzy controller,” *Fuzzy Sets Syst.*, vol. 62, pp. 135–141, 1994.
[8] R. Langari and M. Tomizuka, “Stability of fuzzy linguistic control systems,” in *Proc. IEEE Conf. Decision Contr.*, HI, 1990.
[9] R. Langari, “A nonlinear formulation of a class of fuzzy linguistic control algorithms,” in *Proc. Amer. Contr. Conf.*, Green Valley, AZ, pp. 2273–22781992.
[10] C. C. Lee, “Fuzzy logic in control system: Fuzzy logic controller—Parts I and II,” *IEEE Trans. Syst., Man, .Cybern.*, vol. 20, pp. 408–435, 1990.
[11] F. L. Lewis and K. Liu, “Toward a paradigm for fuzzy logic control,” *Automatica*, vol. 32, pp. 167–181, 1996.
[12] H. A. Malki, H. D. Li, and G. Chen, “New design and stability analysis of fuzzy PF controllers,” *IEEE Trans. Fuzzy Syst.*, vol. 2, pp. 245–254, 1994.
[13] F. Matia, A. Jimenez, R. Galan, and R. Sanz, “Fuzzy controllers: Lifting the linear–nonlinear frontier,” *Fuzzy Sets Syst.*, vol. 52, pp. 113–129, 1992.
[14] J. M. Mendel, “Fuzzy logic systems for engineering: A tutorial,” *Proc. IEEE*, vol. 83, pp. 345–377, 1995.
[15] K. Ogata, *System Dynamics*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1992.

[16] Y. M. Pok and J. X. Xu, "An analysis of fuzzy control systems using vector space," in *Proc. 2nd IEEE Int. Conf. Fuzzy Syst.*, San Francisco, CA, Mar. 28–Apr. 1, 1993, pp. 363–368.
[17] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
[18] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, pp. 116–132, 1985.
[19] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H^{∞} control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, pp. 1–13, Feb. 1996.
[20] K. L. Tang and R. J. Mulholland, "Comparing fuzzy logic with classical controller designs," *IEEE Trans. Syst., Man, Cybern.*, vol. 17, pp. 1085–1087, 1987.
[21] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issue," *IEEE Trans. Fuzzy Syst.*, vol. 4, pp. 14–23, Feb. 1996.
[22] L. Wang, *Adaptive Fuzzy Systems and Control—Design and Stability Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1994.
[23] P. Z. Wang, H. M. Zhang, and W. Xu, "Pad-analysis of fuzzy control stability," *Fuzzy Sets Syst.*, vol. 38, pp. 27–42, 1990.
[24] C. C. Wong, C. H. Chou, and D. L. Mon, "Studies on the output of fuzzy controller with multiple inputs," *Fuzzy Sets Syst.*, vol. 57, pp. 149–158, 1993.
[25] R. R. Yager and D. P. Filev, *Essentials of Fuzzy Modeling and Control*. New York: Wiley, 1994.
[26] J. Yen, R. Langari, and L. A. Zadeh, Eds., *Industrial Applications of Fuzzy Control and Intelligent Systems*. New York: IEEE Press, 1995.
[27] H. Ying, W. Siler, and J. J. Buckley, "Fuzzy control theory: A nonlinear case," *Automatica*, vol. 26, pp. 513–520, 1990.
[28] H. Ying, "The simplest fuzzy controllers using different inference methods are different nonlinear proportional-integral controllers with variable gains," *Automatica*, vol. 29, pp. 1579–1589, 1993.
[29] ——, "A fuzzy controller with linear control rules is the sum of a global two-dimensional multilevel relay and a local nonlinear proportional-integral controller," *Automatica*, vol. 29, pp. 499–505, 1993.
[30] ——, "Practical design of nonlinear fuzzy controllers with stability analysis for regulating processes with unknown mathematical models," *Automatica*, vol. 30, pp. 1185–1195, 1994.
[31] ——, "Analytical structure of a two-input two-output fuzzy controller and its relation to PI and multilevel relay controllers," *Fuzzy Sets Syst.*, vol. 63, pp. 21–33, 1994.
[32] H. Ying and L. C. Sheppard, "Analytical structure of Takagi–Sugeno fuzzy PID controllers and their applications in control of mean arterial pressure in patients," in *Proc. 5th Eur. Congress Intell. Tech. Soft Computing*, Aachen, Germany, Sept. 1997, pp. 1374–1378.
[33] H. Ying, "The Takagi–Sugeno fuzzy controllers using the simplified linear control rules are nonlinear variable gain controllers," *Automatica*, to be published.
[34] ——, "General Takagi–Sugeno fuzzy systems with simplified linear rule consequent are universal controllers, models, and filters," *Inform. Sci.*, to be published.

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