

Necessary Conditions on Minimal System Configuration for General MISO Mamdani Fuzzy Systems as Universal Approximators

Yongsheng Ding, *Associate Member, IEEE*, Hao Ying, *Senior Member, IEEE*, and Shihuang Shao

Abstract—Recent studies have shown that both Mamdani-type and Takagi-Sugeno-type fuzzy systems are universal approximators in that they can uniformly approximate continuous functions defined on compact domains with arbitrarily high approximation accuracy. In this paper, we investigate necessary conditions for general multiple-input single-output (MISO) Mamdani fuzzy systems as universal approximators with as minimal system configuration as possible. The general MISO fuzzy systems employ almost arbitrary continuous input fuzzy sets, arbitrary singleton output fuzzy sets, arbitrary fuzzy rules, product fuzzy logic AND, and the generalized defuzzifier containing the popular centroid defuzzifier as a special case. Our necessary conditions are developed under the practically sensible assumption that only a finite set of extrema of the multivariate continuous function to be approximated is available. We have first revealed a decomposition property of the general fuzzy systems: A r -input fuzzy system can always be decomposed to the sum of r simpler fuzzy systems where the first system has only one input variable, the second one two input variables, and the last one r input variables. Utilizing this property, we have derived some necessary conditions for the fuzzy systems to be universal approximators with minimal system configuration. The conditions expose the strength as well as limitation of the fuzzy approximation: 1) only a small number of fuzzy rules may be needed to uniformly approximate multivariate continuous functions that have a complicated formulation but a relatively small number of extrema; and 2) the number of fuzzy rules must be large in order to approximate highly oscillatory continuous functions. A numerical example is given to demonstrate our new results.

I. INTRODUCTION

FUZZY systems have successfully been used for a variety of applications, especially in control applications where human expert control strategy is emulated and in modeling applications where behavior of physical systems is represented. From a mathematics standpoint, fuzzy systems are just practical function approximators. In the past few years, various fuzzy systems have been proved to be universal approximators in that they can uniformly approximate any continuous functions defined on compact domains to any degree of accuracy ([1]–[4], [6], [7], [9]–[13], [15]–[20]). We have constructively proved that the general multiple-input single-output (MISO) fuzzy sys-

tems, Mamdani-type or Takagi-Sugeno type, are universal approximators, and have established some sufficient conditions for uniform approximation with a pre-specified approximation accuracy [13], [15]–[17]. Approximation accuracy and properties of some fuzzy systems have also been analyzed and compared [20], [21].

Approximation quality of a fuzzy system depends on its system configuration, that is, the selection of input fuzzy sets, output fuzzy sets, fuzzy rules, inference methods, and defuzzifiers. Approximation quality also depends on the characteristics of the function to be approximated. Given a continuous function, function approximation by the fuzzy systems can always be achieved if the number of fuzzy sets and fuzzy rules is allowed to increase as large as needed [13], [15]–[17]. However, achieving better approximation at the expense of a larger number of fuzzy sets and fuzzy rules is not desirable both in theory and in practice. One always wants to approximate a given function with as a simple system configuration as possible. Thus, the central and practical question is: What are the necessary conditions under which fuzzy systems can possibly be universal approximators but with as minimal system configuration as possible? We have established such necessary conditions for the general single-input single-output (SISO) Mamdani fuzzy systems and a MISO Mamdani fuzzy system that uses triangular membership functions and linear fuzzy rules [14]. In this paper, we will establish some necessary conditions for a much more general class of MISO Mamdani fuzzy systems that are universal approximators as we have proved in [13]. We will use the resulting necessary conditions and a numerical example to explore and demonstrate, in a mathematically rigorous way, the strength and limitation of the general fuzzy approximators. The work in this paper is new and general and has not been presented in our previous papers.

II. CONFIGURATION OF GENERAL MISO FUZZY SYSTEMS

There are r independent input variables represented by an input variable vector

$$\underline{x} = (x_1, x_2, \dots, x_r)$$

where the domain of x_i is $\Theta_i = [a_i, b_i]$ and that of \underline{x} is

$$\Theta = \Theta_1 \times \Theta_2 \times \dots \times \Theta_r = [a_1, b_1] \times [a_2, b_2] \times \dots \times [a_r, b_r].$$

The interval $[a_i, b_i]$ is divided into N_i subintervals:

$$a_i = C_0^i < C_1^i < C_2^i < \dots < C_{N_i-1}^i < C_{N_i}^i = b_i, \\ 1 \leq i \leq r.$$

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Y. Ding and S. Shao are with the Department of Electrical Engineering, Dong Hua University, Shanghai, 200051, China.

H. Ying is with the Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202 USA.

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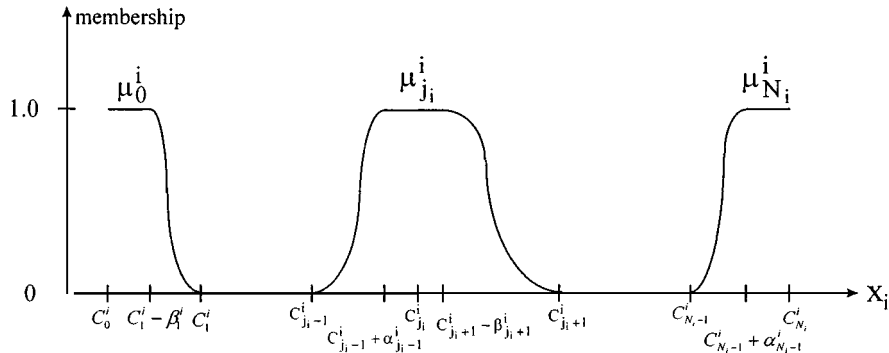


Fig. 1. Illustrative definition of input fuzzy sets for the general MISO fuzzy systems.

On Θ_i , $N_i + 1$ convex, normal, continuous input fuzzy sets, denoted as $A_{j_i}^i$ ($0 \leq j_i \leq N_i$), are defined to fuzzify x_i . $A_{j_i}^i$ has a membership function, designated as $\mu_{j_i}^i$, whose value is zero at $x_i = C_{j_i-1}^i$, and increases monotonically on $[C_{j_i-1}^i, C_{j_i-1}^i + \alpha_{j_i-1}^i]$ where $0 < \alpha_{j_i-1}^i \leq C_{j_i}^i - C_{j_i-1}^i$, and reaches one at $x_i = C_{j_i-1}^i + \alpha_{j_i-1}^i$. $\mu_{j_i}^i$ is one on $[C_{j_i-1}^i + \alpha_{j_i-1}^i, C_{j_i+1}^i - \beta_{j_i+1}^i]$, where $0 < \beta_{j_i+1}^i \leq C_{j_i+1}^i - C_{j_i}^i$, and decreases monotonically and becomes zero at $x_i = C_{j_i+1}^i$. $\mu_{j_i}^i$ is zero when x_i is outside $[C_{j_i-1}^i, C_{j_i+1}^i]$. For the end points $C_0^i = a_i$ and $C_{N_i}^i = b_i$, μ_0^i is one on $[C_0^i, C_1^i - \beta_1^i]$, where $0 < \beta_1^i \leq C_1^i - C_0^i$, and then decreases monotonically to zero at $x_i = C_1^i$. $\mu_{N_i}^i$ is zero at $x_i = C_{N_i-1}^i$ and then increases monotonically to one at $x_i = C_{N_i-1}^i + \alpha_{N_i-1}^i$, where $0 < \alpha_{N_i-1}^i \leq C_{N_i}^i - C_{N_i-1}^i$, and remains to be one until $x_i = C_{N_i}^i$. μ_0^i and $\mu_{N_i}^i$ are zero elsewhere. $\mu_{j_i}^i$ intersects with only $\mu_{j_i-1}^i$ and $\mu_{j_i+1}^i$, and only once. These definitions are illustrated in Fig. 1. The definitions of the input fuzzy sets are very general and contain almost all of the fuzzy sets employed in fuzzy systems [8] (e.g., fuzzy controllers and fuzzy models).

Ω Mamdani fuzzy rules in the following form are used:

$$\begin{aligned} &\text{IF } x_1 \text{ is } A_{h_1}^1 \text{ AND } x_2 \text{ is } A_{h_2}^2 \text{ AND} \\ &\dots \text{ AND } x_r \text{ is } A_{h_r}^r \text{ THEN } y \text{ is } B_{h_1, h_2, \dots, h_r} \end{aligned} \quad (1)$$

where B_{h_1, h_2, \dots, h_r} is a singleton output fuzzy set for the output variable y . B_{h_1, h_2, \dots, h_r} is nonzero only at $y = V_{h_1, h_2, \dots, h_r}$ (an arbitrary constant). Product fuzzy logic AND is employed to evaluate the AND's in the fuzzy rules, and the combined membership for B_{h_1, h_2, \dots, h_r} is $\mu_{h_1}^1 \mu_{h_2}^2 \dots \mu_{h_r}^r$. To obtain the output of the MISO fuzzy systems, we use the generalized defuzzifier [5]

$$F(\underline{x}) \stackrel{\text{def}}{=} y = \frac{\sum (\mu_{h_1}^1 \mu_{h_2}^2 \dots \mu_{h_r}^r)^\alpha \cdot V_{h_1, h_2, \dots, h_r}}{\sum (\mu_{h_1}^1 \mu_{h_2}^2 \dots \mu_{h_r}^r)^\alpha} \quad (2)$$

Different defuzzifier can be obtained by using different α , where $0 \leq \alpha < \infty$. The centroid defuzzifier is obtained when $\alpha = 1$, and the mean of maximum defuzzifier when $\alpha = \infty$.

III. PROBLEM STATEMENT

The MISO Mamdani fuzzy systems configured above are very general and they are universal approximators as we proved previously. The sufficient conditions [13] we have obtained

show that the general fuzzy systems are able to achieve function approximation if the number of fuzzy sets and fuzzy rules is allowed to increase as large as needed. The issue we address in this paper is: What are the necessary conditions under which the general fuzzy systems can possibly be universal approximators but with as minimal system configuration as possible? Mathematically speaking, designate \overline{C}_f as the family of r -input one-output continuous functions, defined on the r -dimensional compact domain Θ , which have a finite number of extrema. Suppose that the following information is available:

- 1) An arbitrarily small approximation error bound $\varepsilon > 0$;
- 2) a set of extrema, $\{m_j\}$, of an arbitrarily selected function $f(\underline{x}) \in \overline{C}_f$, at

$$\underline{x} = \underline{H}_j = (h_j^1, h_j^2, \dots, h_j^r) \in (a_1, b_1) \times (a_2, b_2) \times \dots \times (a_r, b_r), \quad j = 1, 2, \dots, M;$$
- 3) the values of $f(\underline{x})$ at $x_i = a_i$ and $x_i = b_i$ for all $i = 1, 2, \dots, r$ (there are a total of 2^r such values).

Then, the question is: What are the necessary conditions under which there always exists an above-defined r -input one-output fuzzy system whose configuration is as small as possible but still satisfies

$$\max_{\underline{x} \in \Theta} |F(\underline{x}) - f(\underline{x})| \leq \varepsilon? \quad (3)$$

As always, one needs to impose certain assumptions on the continuous function to be approximated before establishing necessary conditions for fuzzy systems, or any other types of approximators. Different assumptions require different amount of information be available. If too much information is required, fuzzy system technology may not be necessary in the first place, since many other well-developed classical functional approximators, such as spline functions, can be used to perform the approximation more efficiently. Bearing this point in mind, we want our assumptions to be as less restrictive as possible, and yet still practically sensible. In this paper, we consider the worst case and assume that only a set of extrema of $f(\underline{x})$ is known, which is probably the minimum amount of information necessary for characterizing major features of a well-behaved continuous function. More importantly, such information can be gained in practice if the continuous function is readily measurable.

IV. MAIN RESULTS

The following lemma deals with the continuity of $F(\underline{x})$ in relation to assignment of fuzzy rules.

Lemma 1: $F(\underline{x})$ is continuous on the entire compact domain Θ if and only if all the 2^r different fuzzy rules in the form of (1) are assigned to each of the $N_1 \times N_2 \times \dots \times N_r$ different combinations of subintervals.

Proof: Without losing generality, we assume $\underline{x} \in [C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$. We first prove the sufficiency of the condition. Due to the way the input fuzzy sets are defined, only two nonzero memberships are resulted for each input variable after fuzzification. They are

$$\begin{aligned} \mu_{j_1}^1 \text{ and } \mu_{j_1+1}^1 & \text{ for } x_1 \\ \mu_{j_2}^2 \text{ and } \mu_{j_2+1}^2 & \text{ for } x_2 \\ \mu_{j_r}^r \text{ and } \mu_{j_r+1}^r & \text{ for } x_r. \end{aligned}$$

Consequently, 2^r fuzzy rules relating to these memberships are fired, and we get the equation shown at the bottom of the page. $(\mu_{j_1+i_1}^1 \mu_{j_2+i_2}^2 \dots \mu_{j_r+i_r}^r)^\alpha$ is a continuous function of \underline{x} on Θ and $V_{j_1+i_1, j_2+i_2, \dots, j_r+i_r}$ is constant, therefore, $F(\underline{x})$ is continuous on Θ .

We now prove the necessity of the condition. When at least one of the 2^r rules is not used, say the following one is not fired:

$$\begin{aligned} \text{IF } x_1 \text{ is } A_{j_1}^1 \text{ AND } x_2 \text{ is } A_{j_2}^2 \text{ AND} \\ \dots \text{ AND } x_r \text{ is } A_{j_r}^r \text{ THEN } y \text{ is } B_{j_1, j_2, \dots, j_r}. \end{aligned} \quad (4)$$

Then, at the point $(C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r)$, all the combinations of the r memberships are zero but $(\mu_{j_1}^1 \mu_{j_2}^2 \dots \mu_{j_r}^r)^\alpha$. However, this membership combination does not exist because the fuzzy rule (4) is not used. As a result, both the numerator and denominator of (2) is zero, meaning $F(\underline{x})$ has no definition at $(C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r)$. So, $F(\underline{x})$ is not continuous on Θ . ■

Lemma 2: Suppose that Θ is divided into $N_1 \times N_2 \times \dots \times N_r$ cubes, $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$ for $j_i = 0, 1, \dots, N_i - 1$, where $i = 1, 2, \dots, r$. $F(\underline{x})$ is continuous on Θ if and only if $\Omega = \prod_{i=1}^r (N_i + 1)$ fuzzy rules in the form of (1) are used.

Proof: According to Lemma 1, for each cube $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$, all the 2^r different combinations of the r memberships should be used to form 2^r fuzzy rules in order to gain the continuity of $F(\underline{x})$ on the cube. This means to ensure the continuity of $F(\underline{x})$ on Θ , all the possible combinations of the r memberships should be used. There are $\prod_{i=1}^r (N_i + 1)$ such membership combinations, resulting the same number of fuzzy rules. ■

Lemma 3: At $\underline{x} = (C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r)$ for all $j_i = 0, 1, \dots, N_i$ where $i = 1, 2, \dots, r$, $F(\underline{x}) = V_{j_1, j_2, \dots, j_r}$.

Proof: At $\underline{x} = (C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r)$, $\mu_{j_1}^i = 1$ and $\mu_{j_i+1}^i = 0$, for all $j_i = 0, 1, \dots, N_i - 1$. Therefore, only the following one fuzzy rule is executed:

$$\begin{aligned} \text{IF } x_1 \text{ is } A_{j_1}^1 \text{ AND } x_2 \text{ is } A_{j_2}^2 \text{ AND} \\ \dots \text{ AND } x_r \text{ is } A_{j_r}^r \text{ THEN } y \text{ is } B_{j_1, j_2, \dots, j_r}. \end{aligned}$$

After using the product fuzzy logic AND, B_{j_1, j_2, \dots, j_r} is assigned a membership value of one and consequently $F(\underline{x}) = V_{j_1, j_2, \dots, j_r}$. At $\underline{x} = (C_{N_1}^1, C_{N_2}^2, \dots, C_{N_r}^r)$, it is straightforward to prove that $F(\underline{x}) = V_{N_1, N_2, \dots, N_r}$. ■

In the following theorem, we reveal a decomposition property of the general MISO fuzzy systems. This theorem is very useful in our proof of Lemma 4 that shows the fuzzy systems are monotonic in each of the cubes configured by r subintervals. The monotonicity is the key for the establishment of the necessary conditions stated in Theorem 2.

Theorem 1: (Decomposition of the general MISO Mamdani fuzzy systems)

A r -input general fuzzy system can always be decomposed to the sum of r simpler fuzzy systems: The first system has one input variable, the second one two input variables and the last one r input variables. There exist $r!$ different such decompositions because there are $r!$ different arrangements of the input variables in the sum. One of the $r!$ decompositions is:

$$\begin{aligned} F(\underline{x}) = R_1(x_1) + Q_1(x_1)\varphi_2(x_2) + Q_2(x_1, x_2)\varphi_3(x_3) + \dots \\ + Q_{r-2}(x_1, \dots, x_{r-2})\varphi_{r-1}(x_{r-1}) \\ + Q_{r-1}(x_1, \dots, x_{r-1})\varphi_r(x_r) \end{aligned}$$

where

$$\varphi_i(x_i) = \frac{(\mu_{j_i+1}^i)^\alpha}{(\mu_{j_i}^i)^\alpha + (\mu_{j_i+1}^i)^\alpha}, \quad i = 1, 2, \dots, r.$$

Proof: We do first level decomposition as follows:

$$F(\underline{x}) = R_{r-1}(x_1, \dots, x_{r-1}) + Q_{r-1}(x_1, \dots, x_{r-1})\varphi_r(x_r)$$

where

$$\varphi_r(x_r) = \frac{(\mu_{j_r+1}^r)^\alpha}{(\mu_{j_r}^r)^\alpha + (\mu_{j_r+1}^r)^\alpha}$$

(see equation at the bottom of the page.) We then carry out second level decomposition by decomposing $R_{r-1}(x_1, \dots, x_{r-1})$ in a similar fashion:

$$\begin{aligned} R_{r-1}(x_1, \dots, x_{r-1}) = R_{r-2}(x_1, \dots, x_{r-2}) \\ + Q_{r-2}(x_1, \dots, x_{r-2})\varphi_{r-1}(x_{r-1}) \end{aligned}$$

where

$$\varphi_{r-1}(x_{r-1}) = \frac{(\mu_{j_{r-1}+1}^{r-1})^\alpha}{(\mu_{j_{r-1}}^{r-1})^\alpha + (\mu_{j_{r-1}+1}^{r-1})^\alpha}$$

$$F(\underline{x}) = \frac{\sum_{i_1=0}^1 \sum_{i_2=0}^1 \dots \sum_{i_r=0}^1 (\mu_{j_1+i_1}^1 \mu_{j_2+i_2}^2 \dots \mu_{j_r+i_r}^r)^\alpha \cdot V_{j_1+i_1, j_2+i_2, \dots, j_r+i_r}}{\sum_{i_1=0}^1 \sum_{i_2=0}^1 \dots \sum_{i_r=0}^1 (\mu_{j_1+i_1}^1 \mu_{j_2+i_2}^2 \dots \mu_{j_r+i_r}^r)^\alpha}.$$

(see equation at the bottom of the page.) Note that the expressions of $\varphi_r(x_r)$ and $\varphi_{r-1}(x_{r-1})$ are identical and the only difference is the input variable involved. Combining the preceding two decompositions, we get

$$F(\underline{x}) = R_{r-2}(x_1, \dots, x_{r-2}) + Q_{r-2}(x_1, \dots, x_{r-2})\varphi_{r-1}(x_{r-1}) \\ + Q_{r-1}(x_1, \dots, x_{r-1})\varphi_r(x_r).$$

We can keep decomposing to the r th level, which gives us

$$F(\underline{x}) = R_1(x_1) + Q_1(x_1)\varphi_2(x_2) + Q_2(x_1, x_2)\varphi_3(x_3) + \dots \\ + Q_{r-2}(x_1, \dots, x_{r-2})\varphi_{r-1}(x_{r-1}) \\ + Q_{r-1}(x_1, \dots, x_{r-1})\varphi_r(x_r),$$

where

$$R_1(x_1) = \frac{(\mu_{j_1}^1)^\alpha V_{j_1, j_2, \dots, j_r} + (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2, \dots, j_r}}{(\mu_{j_1}^1)^\alpha + (\mu_{j_1+1}^1)^\alpha}$$

(see equation at the bottom of the page.) We should point out that the decomposition is nonunique due to the existence of dif-

ferent arrangements of the r input variables. For example, the following is another decomposition:

$$F(\underline{x}) = T_1(x_r) + S_1(x_r)\varphi_{r-1}(x_{r-1}) \\ + S_2(x_{r-1}, x_r)\varphi_{r-2}(x_{r-2}) + \dots \\ + S_{r-2}(x_3, \dots, x_r)\varphi_2(x_2) + S_{r-1}(x_2, \dots, x_r)\varphi_1(x_1).$$

Because there are total $r!$ different kinds of arrangements of the r input variables, there are the same number of different decompositions. Importantly though, all these decompositions have the same property: A r -input general fuzzy system is decomposed to the sum of r simpler fuzzy systems with the first system having one input variable, the second one two input variables, and the last one r input variables. ■

Lemma 4: When the conditions set in Lemma 1 are met, $F(\underline{x})$ is monotonic on $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$ for $j_i = 0, 1, \dots, N_i - 1$, where $i = 1, 2, \dots, r$.

Proof: For better presentation, we will only prove $r = 2$ case here. The results can be extended to higher dimension. Without losing generality, we assume $x_1 \in [C_{j_1}^1, C_{j_1+1}^1]$ and

$$R_{r-1}(x_1, \dots, x_{r-1}) = \frac{\sum_{i_1=0}^1 \dots \sum_{i_{r-1}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-1}+i_{r-1}}^{r-1})^\alpha \cdot V_{j_1+i_1, \dots, j_{r-1}+i_{r-1}, j_r}}{\sum_{i_1=0}^1 \dots \sum_{i_{r-1}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-1}+i_{r-1}}^{r-1})^\alpha}$$

$$Q_{r-1}(x_1, \dots, x_{r-1}) = \frac{\sum_{i_1=0}^1 \dots \sum_{i_{r-1}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-1}+i_{r-1}}^{r-1})^\alpha (V_{j_1+i_1, \dots, j_{r-1}+i_{r-1}, j_r+1} - V_{j_1+i_1, \dots, j_{r-1}+i_{r-1}, j_r})}{\sum_{i_1=0}^1 \dots \sum_{i_{r-1}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-1}+i_{r-1}}^{r-1})^\alpha}.$$

$$R_{r-2}(x_1, \dots, x_{r-2}) = \frac{\sum_{i_1=0}^1 \dots \sum_{i_{r-2}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-2}+i_{r-2}}^{r-2})^\alpha \cdot V_{j_1+i_1, \dots, j_{r-2}+i_{r-2}, j_{r-1}, j_r}}{\sum_{i_1=0}^1 \dots \sum_{i_{r-2}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-2}+i_{r-2}}^{r-2})^\alpha},$$

$$Q_{r-2}(x_1, \dots, x_{r-2}) = \frac{\sum_{i_1=0}^1 \dots \sum_{i_{r-2}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-2}+i_{r-2}}^{r-2})^\alpha (V_{j_1+i_1, \dots, j_{r-2}+i_{r-2}, j_{r-1}+1, j_r} - V_{j_1+i_1, \dots, j_{r-2}+i_{r-2}, j_{r-1}, j_r})}{\sum_{i_1=0}^1 \dots \sum_{i_{r-2}=0}^1 (\mu_{j_1+i_1}^1 \dots \mu_{j_{r-2}+i_{r-2}}^{r-2})^\alpha}.$$

$$Q_1(x_1) = \frac{(\mu_{j_1}^1)^\alpha V_{j_1, j_2+1, j_3, \dots, j_r} + (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2+1, j_3, \dots, j_r} - (\mu_{j_1}^1)^\alpha V_{j_1, j_2, \dots, j_r} - (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2, \dots, j_r}}{(\mu_{j_1}^1)^\alpha + (\mu_{j_1+1}^1)^\alpha}.$$

$x_2 \in [C_{j_2}^2, C_{j_2+1}^2]$. Using the decomposition property stated in Theorem 1, we have

$$F(x_1, x_2) = R(x_1) + Q(x_1)\varphi_2(x_2)$$

where

$$R(x_1) = \frac{(\mu_{j_1}^1)^\alpha V_{j_1, j_2} + (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2}}{(\mu_{j_1}^1)^\alpha + (\mu_{j_1+1}^1)^\alpha}$$

(see equation at the bottom of the page)

$$\varphi_2(x_2) = \frac{(\mu_{j_2+1}^2)^\alpha}{(\mu_{j_2}^2)^\alpha + (\mu_{j_2+1}^2)^\alpha}.$$

We now prove that $F(x_1, x_2)$ does not have extrema on $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2]$, which means $F(x_1, x_2)$ is monotonic. Note that

$$\begin{aligned} \frac{\partial F}{\partial x_1} &= \frac{dR}{dx_1} + \frac{dQ}{dx_1} \varphi_2(x_2) \\ \frac{\partial^2 F}{\partial x_1^2} &= \frac{d^2 R}{dx_1^2} + \frac{d^2 Q}{dx_1^2} \varphi_2(x_2) \\ \frac{\partial F}{\partial x_2} &= Q(x_1) \frac{d\varphi_2}{dx_2} \\ \frac{\partial^2 F}{\partial x_2^2} &= Q(x_1) \frac{d^2 \varphi_2}{dx_2^2} \\ \frac{\partial^2 F}{\partial x_1 \partial x_2} &= \frac{\partial^2 F}{\partial x_2 \partial x_1} = \frac{dQ}{dx_1} \frac{d\varphi_2}{dx_2}. \end{aligned}$$

The necessary conditions for $F(x_1, x_2)$ to have extrema are $(\partial F/\partial x_1) = 0$ and $(\partial F/\partial x_2) = 0$. Because on $[C_{j_2}^2, C_{j_2+1}^2]$, $(d\mu_{j_2+1}^2/dx_2) \geq 0$ and $(d\mu_{j_2}^2/dx_2) \leq 0$, and the both cannot be zero at the same time, we have

$$\frac{d\varphi_2}{dx_2} = \frac{\alpha(\mu_{j_2}^2 \mu_{j_2+1}^2)^{\alpha-1} \left[\mu_{j_2}^2 \frac{d\mu_{j_2+1}^2}{dx_2} - \mu_{j_2+1}^2 \frac{d\mu_{j_2}^2}{dx_2} \right]}{[(\mu_{j_2}^2)^\alpha + (\mu_{j_2+1}^2)^\alpha]^2} \neq 0.$$

Thus, the only possibility that $(\partial F/\partial x_2) = 0$ is when $Q(x_1) = 0$, which results $(\partial^2 F/\partial x_2^2) = 0$ and hence

$$\begin{aligned} D &= \begin{vmatrix} \frac{\partial^2 F}{\partial x_1^2} & \frac{\partial^2 F}{\partial x_1 \partial x_2} \\ \frac{\partial^2 F}{\partial x_2 \partial x_1} & \frac{\partial^2 F}{\partial x_2^2} \end{vmatrix} \\ &= -\frac{\partial^2 F}{\partial x_1 \partial x_2} \times \frac{\partial^2 F}{\partial x_2 \partial x_1} = -\left[\frac{dQ}{dx_1} \times \frac{d\varphi_2}{dx_2} \right]^2. \end{aligned}$$

D becomes zero only when $(dQ/dx_1) = 0$. When $(dQ/dx_1) = 0$, $(\partial F/\partial x_1) = (dR/dx_1) = 0$, leading to $V_{j_1, j_2} = V_{j_1+1, j_2}$, which in turn means

$$F(x_1, x_2) = V_{j_1, j_2} = V_{j_1+1, j_2}.$$

That is, $F(x_1, x_2)$ becomes a constant and does not have extrema.

Thus, $D < 0$ on $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2]$ and consequently $F(x_1, x_2)$ has no extrema, or equivalently, is monotonic. Extending this analysis to more than two input variables, we conclude that $F(\underline{x})$ is monotonic on $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$.

Recall that when formulating (3), we assumed that M extrema of $f(\underline{x})$, at $\underline{x} = \underline{H}_j = (h_j^1, h_j^2, \dots, h_j^r)$, where $j = 1, 2, \dots, M$, are given. Without loss of generality, we assume that, for a fixed $1 \leq i \leq r$, we have M points inside the interval $[a_i, b_i]$ but not at the two end points: $S_i = (h_1^i, h_2^i, \dots, h_M^i)$. We will only keep the distinct points in S_i and rearrange them in the ascendant order to form the following new sets:

$$T_i = (\theta_1^i, \theta_2^i, \dots, \theta_{K_i}^i), \quad i = 1, 2, \dots, r \quad (5)$$

where $\theta_1^i < \theta_2^i < \dots < \theta_{K_i}^i$. Here, we suppose that T_i has K_i ($1 \leq K_i \leq M$) distinct points. Obviously, T_i divides $[a_i, b_i]$ into $K_i + 1$ subintervals.

Now, using the notation T_i defined in (5) and the lemmas on the general MISO fuzzy systems, we establish the following necessary conditions for the general MISO fuzzy systems as universal function approximators with minimal system configuration.

Theorem 2: Given the distinct points K_i ($i = 1, 2, \dots, r$) on $[a_i, b_i]$, the following necessary conditions must be satisfied simultaneously in order for the general MISO Mamdani fuzzy systems to achieve the approximation (3) with minimal system configuration:

- (a) For $i = 1, 2, \dots, r$, the interval $[a_i, b_i]$ must be divided into at least $K_i + 1$ subintervals, that is, $N_i \geq K_i + 1$.
- (b) $K_i + 1$ of the N_i subintervals must be formed such that $C_0^i = a_i, C_j^i = \theta_j^i$ ($1 \leq j \leq K_i$), and $C_{K_i+1}^i = b_i$.
- (c) 2^r fuzzy rule of (1) must be assigned to each of the r -dimensional cubes, $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$ for all $j_i = 0, 1, \dots, K_i$. That is, the total number of fuzzy rules is $\Omega = \prod_{i=1}^r (K_i + 2)$.
- (d) The singleton output fuzzy set B_{j_1, j_2, \dots, j_r} must be so defined that it satisfies

$$m_{j_1, j_2, \dots, j_r} - \varepsilon \leq V_{j_1, j_2, \dots, j_r} \leq m_{j_1, j_2, \dots, j_r} + \varepsilon.$$

Proof: We will only prove $r = 2$ case here for condition (a) and the cases with more input variables can be proved in a similar fashion. We use contradiction argument to show that $N_i \geq K_i + 1$ ($i = 1, 2$) is necessary. Suppose that $N_i < K_i + 1$. Let the partition of $[a_i, b_i]$ be

$$a_i = C_0^i < C_1^i < C_2^i < \dots < C_{N_i-1}^i < C_{N_i}^i = b_i.$$

Then there must exist at least one two-dimensional rectangle, say $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2]$, on which the function $f(\underline{x})$ is nonmonotonic. That is, $f(\underline{x})$ has at least one extremum inside this rectangle but not at the four vertexes of the rectangle. Without losing generality, let us assume that there is one maximum, m_h ($1 \leq h \leq M$), at $\underline{x} = (\lambda_{j_1}^1, \lambda_{j_2}^2)$ where $x_i = \lambda_{j_i}^i \in$

$$Q(x_1) = \frac{(\mu_{j_1}^1)^\alpha V_{j_1, j_2+1} + (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2+1} - (\mu_{j_1}^1)^\alpha V_{j_1, j_2} - (\mu_{j_1+1}^1)^\alpha V_{j_1+1, j_2}}{(\mu_{j_1}^1)^\alpha + (\mu_{j_1+1}^1)^\alpha}$$

$(C_{j_i}^i, C_{j_i+1}^i)$. Suppose, for an arbitrarily small approximation error bound $\varepsilon > 0$, the following inequality holds:

$$\max_{\underline{x} \in [C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2]} |F(\underline{x}) - f(\underline{x})| \leq \varepsilon.$$

This implies that the following three inequalities must hold simultaneously:

$$|F(C_{j_1}^1, C_{j_2}^2) - f(C_{j_1}^1, C_{j_2}^2)| \leq \varepsilon \quad (6)$$

$$|F(\lambda_{j_1}^1, \lambda_{j_2}^2) - m_h| \leq \varepsilon \quad (7)$$

$$|F(C_{j_1}^1, C_{j_2+1}^2) - f(C_{j_1+1}^1, C_{j_2+1}^2)| \leq \varepsilon. \quad (8)$$

However, that m_h is a maximum means $m_h > f(C_{j_1}^1, C_{j_2}^2)$ and $m_h > f(C_{j_1+1}^1, C_{j_2+1}^2)$. Hence, loosely speaking, $f(\underline{x})$ increases monotonically from $(C_{j_1}^1, C_{j_2}^2)$ to $(\lambda_{j_1}^1, \lambda_{j_2}^2)$ and then decreases monotonically from $(\lambda_{j_1}^1, \lambda_{j_2}^2)$ to $(C_{j_1+1}^1, C_{j_2+1}^2)$. According to Lemma 4, $F(\underline{x})$ is monotonic on $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2]$. Inequalities (6)-(8) cannot be true simultaneously. This contradiction means that for $i = 1, 2$, the interval $[a_i, b_i]$ must be divided into at least $K_i + 1$ subintervals, that is, $N_i \geq K_i + 1$, which is the necessary condition (a).

Furthermore, according to the above analysis, when $N_i \geq K_i + 1$, the subintervals must be divided in such a way that $f(\underline{x})$ reaches its M extrema only at $\underline{x} = \underline{H}_i$ ($i = 1, 2, \dots, M$). That requires $K_i + 1$ of the N_i subintervals must be formed such that $C_0^i = a_i, C_j^i = \theta_j^i$ ($1 \leq j \leq K_i$), and $C_{K_i}^i = b_i$, which is necessary condition (b).

Now let us analyze the necessity of condition (c). According to Lemma 2, it is necessary to assign 2^r fuzzy rules in the form of (1) to each r -dimensional cube $[C_{j_1}^1, C_{j_1+1}^1] \times [C_{j_2}^2, C_{j_2+1}^2] \times \dots \times [C_{j_r}^r, C_{j_r+1}^r]$, for $j_i = 0, 1, \dots, K_i$, where $i = 1, 2, \dots, r$, to ensure the continuity of $F(\underline{x})$ on each cube. Consequently, the total number of fuzzy rules needed to ensure the continuity of $F(\underline{x})$ on Θ is $\prod_{i=1}^r (K_i + 2)$.

Finally, we show the necessity of condition (d). To realize the approximation (3), the following inequality must be satisfied:

$$\begin{aligned} & |F(C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r) - f(C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r)| \\ & = |F(C_{j_1}^1, C_{j_2}^2, \dots, C_{j_r}^r) - m_{j_1, j_2, \dots, j_r}| \leq \varepsilon \end{aligned} \quad (9)$$

for all the M extrema. According to Lemma 3, (9) can be rewritten as

$$|V_{j_1, j_2, \dots, j_r} - m_{j_1, j_2, \dots, j_r}| \leq \varepsilon$$

or

$$m_{j_1, j_2, \dots, j_r} - \varepsilon \leq V_{j_1, j_2, \dots, j_r} \leq m_{j_1, j_2, \dots, j_r} + \varepsilon.$$

From Theorem 2, one can see that a selection of V_{j_1, j_2, \dots, j_r} of the singleton output fuzzy set B_{j_1, j_2, \dots, j_r} directly depends on the given approximation error bound ε . The smaller the ε , the narrower the range of the value of V_{j_1, j_2, \dots, j_r} . As a limit, $V_{j_1, j_2, \dots, j_r} = m_{j_1, j_2, \dots, j_r}$ for all $j_i = 1, 2, \dots, N_i - 1$, where $i = 1, 2, \dots, r$, when $\varepsilon = 0$.

Theorem 2 sheds some light on the strength and limitation of the general MISO fuzzy systems as universal approximators. We observe that N_i relates to $K_i + 1$ in a certain way. Specifically,

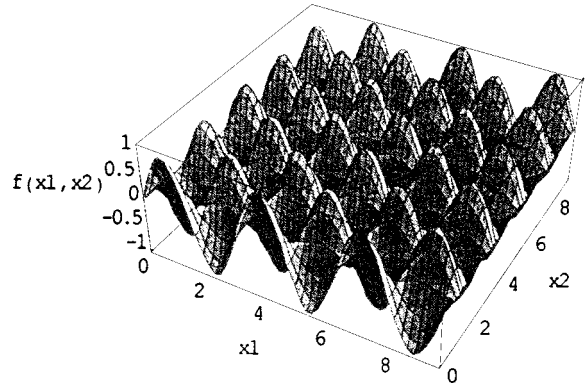


Fig. 2. Graphical illustration of a simple but highly-oscillatory function $f(x_1, x_2) = \sin(2x_1)\cos(3x_2)$ on $[0, 3\pi] \times [0, 3\pi]$ which has 48 extrema on $(0, 3\pi) \times (0, 3\pi)$.

according to Theorem 2, the minimal number of needed fuzzy rules, Ω , increases with the increase of the number of distinct points K_i at which the continuous function gets extreme values. Therefore, if K_i is a small number, Ω can be a small number. Better yet, Ω does not relate to the approximation error bound ε . These facts suggest that, even if a given ε is very small, a small number of fuzzy rules may suffice to uniformly approximate those continuous functions that have a complicated formulation but a relatively small number of extrema. This insightful analysis offers an explanation to the fact that the majority of practical fuzzy controllers and fuzzy models only used a small number of fuzzy rules to achieve the successful applications. On the other hand, the limitation of the fuzzy systems is also exposed by the fact that the number of fuzzy rules needed increases with the increase of the number of extrema of the continuous function. A large number of fuzzy rules is necessary for uniform approximation of functions that are simple but have a lot of extrema. This means that the fuzzy systems are not ideal function approximators for highly-oscillatory functions.

V. AN ILLUSTRATIVE EXAMPLE

Example: What is the minimal number of fuzzy rules and fuzzy sets needed for the general MISO fuzzy systems to approximate $f(x_1, x_2) = \sin(mx_1)\cos(nx_2)$ where m and n are positive integers, defined on $[0, 3\pi] \times [0, 3\pi]$?

Solution: For visualization of the function, we plot $f(x_1, x_2) = \sin(2x_1)\cos(3x_2)$ in Fig. 2. To determine how many extrema the function has, we do the following:

$$\frac{\partial f}{\partial x_1} = m \cos(mx_1) \cos(nx_2)$$

$$\frac{\partial^2 f}{\partial x_1^2} = -m^2 \sin(mx_1) \cos(nx_2)$$

$$\frac{\partial f}{\partial x_2} = -n \sin(mx_1) \sin(nx_2)$$

$$\frac{\partial^2 f}{\partial x_2^2} = -n^2 \sin(mx_1) \cos(nx_2)$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial^2 f}{\partial x_2 \partial x_1} = -mn \cos(mx_1) \sin(nx_2).$$

Consequently,

$$D = \begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{vmatrix} \\ = m^2 n^2 [\sin^2(mx_1) \cos^2(nx_2) - \cos^2(mx_1) \sin^2(nx_2)].$$

By letting, $(\partial f/\partial x_1) = 0$ and $(\partial f/\partial x_2) = 0$, we find that

$$x_1 = \frac{2k_1\pi + \pi}{2m}, \quad (k_1 = 0, 1, \dots, 3m - 1) \\ x_2 = \frac{k_2\pi}{n}, \quad (k_2 = 1, \dots, 3n - 1)$$

at which $D > 0$, meaning $f(x_1, x_2)$ has a total of $3m(3n - 1)$ extrema on $(0, 3\pi) \times (0, 3\pi)$. Even if m or/and n are moderate, a large number of fuzzy rules is needed for the approximation, according to the necessary conditions. For example, if $m = 10$ and $n = 12$ then $f(x_1, x_2)$ has 1050 extrema on $(0, 3\pi) \times (0, 3\pi)$. As a result, at least 32 and 37 fuzzy sets are necessary for x_1 and x_2 , respectively. These result 1184 fuzzy rules needed by the fuzzy systems to approximate the function. Through this example, one sees that a large number of fuzzy rules is necessary in order to approximate a simple but highly-oscillatory function like $f(x_1, x_2) = \sin(mx_1) \cos(nx_2)$.

VI. CONCLUSIONS

The MISO Mamdani fuzzy systems studied in this paper are general: They employ arbitrary fuzzy rules, almost arbitrary continuous input fuzzy sets, arbitrary singleton output fuzzy sets, product fuzzy logic AND and the generalized defuzzifier containing the widely-used centroid defuzzifier as a special case. We have first revealed a decomposition property of the general fuzzy system: A r -input fuzzy system can always be decomposed to the sum of r simpler fuzzy systems where the first system has only one input variable, the second one two input variables and the last one r input variables. This decomposition property facilitated us to derive some necessary conditions for the general MISO fuzzy systems as universal approximators with minimal system configuration. Based on a given approximation error bound and the extrema of the multivariate continuous function to be approximated, the necessary conditions set the requirements on input fuzzy sets, output fuzzy sets and fuzzy rules. These conditions significantly generalize the ones we obtained for the general SISO fuzzy systems and a MISO fuzzy systems using triangular membership functions and linear fuzzy rules.

The necessary conditions provide insight to the strength as well as limitation of the general MISO fuzzy systems as universal approximators:

- 2) Only a small number of fuzzy rules may be needed to uniformly approximate multivariate continuous functions that have a complicated formulation but a relatively small number of extrema.
- 3) The number of fuzzy rules must be large in order to approximate highly oscillatory continuous functions.

These two facts are expected to have significant theoretical and practical implications on applications of fuzzy systems, especially in the fields of fuzzy control and fuzzy modeling.

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Yongsheng Ding (S'97–A'99) received the B.S. degree in electrical engineering, the M.S. degree (honorary) in computer applications, and the Ph.D. degree in electrical engineering from Dong Hua University (formerly China Textile University), Shanghai, China, in 1989, 1994, and 1998, respectively.

He was a Visiting Scientist at Biomedical Engineering Center, the University of Texas Medical Branch, Galveston, TX, from 1996 to 1998. He is currently an Associate Professor of Department of Electrical Engineering, Dong Hua University. His research interests include fuzzy control, intelligent control, genetic algorithms, DNA computing, artificial immune systems, intelligent Internet information retrieval, mobile agents, ATM networks, and industrial and biomedical applications.

Dr. Ding was one of the five Best Student Paper Competition Finalists at the 1997 IEEE International Conference on Systems, Man, and Cybernetics, Orlando, FL. He became a Shanghai Dawn Scholar in 1999.



Hao Ying (SM'97) received the B.S. and M.S. degrees in electrical engineering from Dong Hua University, Shanghai, China, in 1982 and 1984, respectively, and the Ph.D. degree in biomedical engineering from The University of Alabama at Birmingham in 1990.

He was with the faculty of The University of Texas Medical Branch at Galveston until August 2000, when he joined the Department of Electrical and Computer Engineering, Wayne State University, as an Associate Professor. He is also an Advisory Professor of Dong Hua University, Shanghai, China. He has published one research monograph/advanced textbook entitled *Fuzzy Control and Modeling: Analytical Foundations and Applications* (Piscataway, NJ: IEEE Press, 2000), and 44 peer-reviewed journal papers. He is a Guest Editor for *Information Sciences*, *International Journal of Intelligent Control and Systems*, and *Acta Automatica Sinica*.

Dr. Ying served as a Program Chair for The International Joint Conference of North American Fuzzy Information Processing Society (NAFIPS) Conference, Industrial Fuzzy Control and Intelligent System Conference, and NASA Joint Technology Workshop on Neural Networks and Fuzzy Logic in 1994. Additionally, he has served as the Publication Chair for the 2000 IEEE International Conference on Fuzzy Systems and as a Program Committee Member for many other international conferences, including the Joint 9th International Fuzzy Systems Association and 20th NAFIPS International Conference to be held in 2001.



Shihuang Shao received the B.S. degree in electrical engineering from Southeast University, Nanjin, China, in 1960.

He was a Visiting Scientist at the University of Maryland, College Park, from 1986 to 1988, and was a Professor and Chairman of the Department of Electrical Engineering at Dong Hua University (formerly China Textile University), Shanghai, from 1988 to 1992. He is currently a Professor of Department of Electrical Engineering, and the President of the same university. His research

interests include fuzzy control, neural networks, genetic algorithms, chaos control, DNA computing, intelligent Internet information retrieval, modeling and control of economic systems, and industrial automation.