Deriving Analytical Input–Output Relationship for Fuzzy Controllers Using Arbitrary Input Fuzzy Sets and Zadeh Fuzzy AND Operator

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***Abstract*—A fuzzy controller uses either Zadeh or product fuzzy** AND **operator, with the former being more frequently used than the latter. We have recently published a novel technique for deriving analytical input–output relation for the fuzzy controllers that use Zadeh** AND **operator and arbitrary trapezoidal input fuzzy sets, including triangular ones as special cases. In this paper, we have developed a general technique based on that technique to cover *arbitrary* types of input fuzzy sets. Moreover, we have established some necessary and sufficient conditions to characterize general relationship between shape of input fuzzy sets and shape of input space divisions, an important and integral issue because analytical relationship differs in different regions of input space. The new technique and the shape relations are applicable to any type of fuzzy controllers (e.g., Mamdani type or Takagi–Sugeno type). The analytical structures that we have derived provide an unprecedented opportunity to insightfully and rigorously examine the advantages and shortcomings of different design choices available for various components of the fuzzy controllers. We have focused on type selection for input fuzzy sets of Mamdani fuzzy controllers. Our preliminary analysis indicates that the fuzzy controllers using trapezoidal fuzzy sets may be understood (and possibly analyzed and designed) more sensibly and easily in the context of conventional control theory than the fuzzy controllers using any other types of fuzzy sets. Our proposition is that trapezoidal fuzzy sets should be the first choice and used most of time. Possible implication for automatic learning of input fuzzy sets via neural networks or genetic algorithms is briefly discussed.**

***Index Terms*—Design, fuzzy control, input–output relation, PID control, structure analysis, structure derivation.**

I. Introduction

COMPARED to conventional control theory, analysis and design of fuzzy controllers are substantially more challenging because of some unique difficulties [15]. One of the fundamental issues is universal unavailability of analytical (i.e., mathematical) input–output relation of fuzzy controllers (except a small number of classes [27]), explicitly or implicitly. Deriving such structure is no easy in most cases. The reality is that many fuzzy controllers are constructed via the "knowledge engineering approach" as opposed to the mathematical approaches exclusively adopted in conventional control. Most fuzzy controllers have been treated and used as black-box controllers in the sense that their analytical structures are unknown.

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Knowing the explicit structure information will enable one to insightfully understand how and why fuzzy control works. Many fuzzy controllers have peculiar and interesting structures (e.g., nonlinear controllers with variable gains) [19], [22], [27]. Availability of the structure information may lead to more effective and less conservative, thus more practical, analysis and design techniques that require less trial-and-error effort and produce better control performance. Development of these techniques can, and should, utilize the well-developed conventional (nonlinear) control theory as fuzzy control problems have already been translated into nonlinear control problems (e.g., [4], [13], [17], [22], and [27]). For example, we utilized the explicit relationship between a class of fuzzy controllers and the conventional PI (and PD) controllers that we derived in [22], [23] to more effectively and practically design the fuzzy control systems. Our new design procedure could systematically and quite accurately determine the values of the scaling factors for the input variables as well as that for the output variable even when a nonlinear system was controlled [24]. Also, local stability of the designed system could be ensured. All these were achieved under the assumption that the mathematical model of the nonlinear system was unknown. A real-world application of fuzzy control of mean arterial pressure in patient was used to demonstrate the effectiveness of the design approach (see also [20] for more detailed information on this medical problem). Another example is that the input–output formulas we derived in [19] were used to design the fuzzy control systems with guaranteed bounded-input–bounded-output (BIBO) stability [4]. The results in this paper are expected to make it possible to more effectively design more fuzzy control systems involving more general fuzzy controllers.

Deriving the analytical structure of a fuzzy controller that uses the product AND operator is not very difficult, regardless of shape of input fuzzy sets. On the other hand, obtaining the analytical structure involving Zadeh fuzzy AND operator is challenging even for triangular input fuzzy sets, which are the simplest fuzzy sets [5], [18]. In 1990, we developed the first technique in the literature to cover Zadeh AND operator and a particular class of symmetric trapezoidal (or triangular) input fuzzy sets [19]. Our method has been widely used (e.g., [2], [3], [6], [10], [11], [14], and [16]). Recently, we have generalized it to cover arbitrary trapezoidal/triangular fuzzy sets [28]. We have now developed a new and general technique applicable to ***any*** type of input fuzzy sets.

In this paper, we also study the relationships between the shape of input space divisions needed due to the use of Zadeh

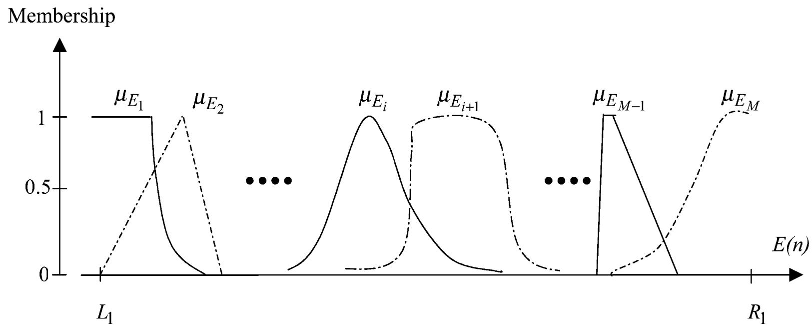

Fig. 1. M example (arbitrary) fuzzy sets of arbitrary types defined for $E(n)$.

AND operator and input fuzzy sets, and we characterize them in the form of necessary and sufficient conditions. Furthermore, we show that trapezoidal input fuzzy sets should be the first choice and used most of time, making design choices simpler. Implication for automatically learned input fuzzy sets for neuro-fuzzy control systems [9], [12] and genetic-algorithm-fuzzy control systems [7], [29] will also be pointed out.

II. Development of a General Technique for Deriving Analytical Input–Output Structure of the Fuzzy Controllers

For better presentation, we will describe the general technique using a class of simpler Mamdani fuzzy controllers. The new technique is actually applicable regardless of the number of input variables and the controller type (i.e., Mamdani type or TS type).

A. A Class of Mamdani Fuzzy Controllers

The controller uses two input variables, $e(n)$ and $r(n)$, and uses system output, $y(n)$, to calculate the scaled error and change of error of $y(n)$ at sampling time n

$$E(n) = K_e e(n) = K_e(SP(n) - y(n))$$
$$R(n) = K_r r(n) = K_r(e(n) - e(n-1))$$

where K_e and K_r are scaling factors, and $SP(n)$ is output command signal. $E(n)$ and $R(n)$ are fuzzified by arbitrary type of input fuzzy sets and a mixture of different types may be used. The fuzzy sets fuzzifying $E(n)$ are defined on the universe $[L_1, R_1]$ whereas the ones fuzzifying $R(n)$ on $[L_2, R_2]$. Fig. 1 illustrates example (arbitrary) fuzzy sets for $E(n)$. $[L_1, R_1]$ is divided into $M-1$ subintervals and M fuzzy sets are defined over them whereas $[L_2, R_2]$ are divided into $N-1$ subintervals where N fuzzy sets are defined. At any sampling time, only two fuzzy sets for $E(n)$ and two fuzzy sets for $R(n)$ are involved in the fuzzification. Without losing generality, we assume that $\widetilde{E}_i$, $\widetilde{E}_{i+1}$, $\widetilde{R}_j$, and $\widetilde{R}_{j+1}$ are the ones that are involved at sampling time n ($\widetilde{E}_i$ and $\widetilde{E}_{i+1}$ for $E(n)$, and $\widetilde{R}_j$ and $\widetilde{R}_{j+1}$ for $R(n)$). Their membership functions are denoted by μ_{E_i}, $\mu_{E_{i+1}}$, μ_{R_j}, and $\mu_{R_{j+1}}$, respectively. Linguistic names may be assigned to them (e.g., positive large and negative small); but this is not necessary in this paper.

Without loss of generality, we assume that graphical definitions of the above four fuzzy sets are as shown in Fig. 2. Their mathematical definitions are

$$\mu_{E_i} = \frac{1}{1+e^{0.3(E(n)-5)}} \qquad -\infty < E(n) < \infty$$
$$\mu_{E_{i+1}} = \begin{cases} 0, & (-\infty, -15] \\ \frac{E(n)+15}{30}, & [-15, 15] \\ 1, & [15, \infty) \end{cases}$$
$$\mu_{R_j} = e^{-((R(n)+15)^2/130)} \qquad -\infty < R(n) < \infty$$
$$\mu_{R_{j+1}} = \frac{1}{1+e^{-0.17(R(n)+2)}}, \qquad -\infty < R(n) < \infty$$

which are defined around $E(n)=0$ and $R(n)=0$. The reader will find later that the new technique works independent of the choice of input fuzzy set types (e.g., shape and location).

The fuzzy controller must use $M \times N$ fuzzy rules to cover all the possible combinations of the input fuzzy sets. However, because only the membership values of $\widetilde{E}_i$, $\widetilde{E}_{i+1}$, $\widetilde{R}_j$, and $\widetilde{R}_{j+1}$ are relevant, only four fuzzy rules are executed as shown in the equation at the bottom of the page, where $u(n)$ is an output variable and $\widetilde{H}_k$, $k = 1, \ldots 4$, is a singleton output fuzzy set whose membership value is nonzero only at $u(n) = h_k$. We place no restriction on the value of h_k in this study. Different inference methods, such as Mamdani minimum inference method, can be used for the fuzzy inference in the rules [22]; the inference results will be the same because the output fuzzy sets are of singleton type. It will become clear that the new technique works if $\widetilde{H}_k$ is any other type of fuzzy set or if the rule consequent uses the TS model. The new technique is also independent of fuzzy inference methods.[1]

[1]A TS type of fuzzy rule with two input variables can be expressed as "IF $E(n)$ is $\widetilde{E}_i$ AND $R(n)$ is $\widetilde{R}_j$, THEN $u(n) = f(e, r)$ where $f()$ can be any linear or nonlinear function.

IF $E(n)$ is $\widetilde{E}_{i+1}$ AND $R(n)$ is $\widetilde{R}_{j+1}$ THEN $u(n)$ is $\widetilde{H}_1$ (Rule 1)
IF $E(n)$ is $\widetilde{E}_{i+1}$ AND $R(n)$ is $\widetilde{R}_j$ THEN $u(n)$ is $\widetilde{H}_2$ (Rule 2)
IF $E(n)$ is $\widetilde{E}_i$ AND $R(n)$ is $\widetilde{R}_{j+1}$ THEN $u(n)$ is $\widetilde{H}_3$ (Rule 3)
IF $E(n)$ is $\widetilde{E}_i$ AND $R(n)$ is $\widetilde{R}_j$ THEN $u(n)$ is $\widetilde{H}_4$ (Rule 4)

Zadeh fuzzy logic AND operator is used to realize the AND operations in the rules. Denoting μ_k the membership value resulted from Zadeh AND operation in Rule k, we have

$$\mu_1 = \min\left(\mu_{\widetilde{E}_{i+1}}, \mu_{\widetilde{R}_{j+1}}\right)$$
$$\mu_2 = \min\left(\mu_{\widetilde{E}_{i+1}}, \mu_{\widetilde{R}_{j}}\right)$$
$$\mu_3 = \min\left(\mu_{\widetilde{E}_{i}}, \mu_{\widetilde{R}_{j+1}}\right)$$
$$\mu_4 = \min\left(\mu_{\widetilde{E}_{i}}, \mu_{\widetilde{R}_{j}}\right)$$

where $\min()$ is the operator taking the lesser of the two membership values as the result.

The popular centroid defuzzifier is employed, which yields

$$U(n) = K_u \cdot u(n) = K_u \frac{\mu_1 h_1 + \mu_2 h_2 + \mu_3 h_3 + \mu_4 h_4}{\mu_1 + \mu_2 + \mu_3 + \mu_4}$$

where K_u is a scaling factor and $U(n)$ is the output of the fuzzy controller at sampling time n.

B. The General Derivation Technique

One key generalization to the original technique in [28] is in the area of input space divisions. This issue is fundamentally important to the development of the new technique. Without its advance, the new technique could have not been developed. For the particular fuzzy controller, the input space is $[L_1, R_1] \times [L_2, R_2]$, which is $(-\infty, \infty) \times (-\infty, \infty)$. Due to the use of Zadeh AND operator, the input space must be divided into a number of regions ***in such a way that in each region a unique analytical inequality relationship can be obtained for each fuzzy rule between the two membership functions being ANDed***. In the case of (arbitrary) trapezoidal or triangular fuzzy sets and two input variables, the division determination involves lines only [28]. (If three input variables are used instead, three-dimensional planes will be involved then). The objective was to determine which line is always smaller over a certain area. The line formula with smaller value is the result of the AND operation whereas the area is an input space division region. This was carried out for an example fuzzy controller in an *ad-hoc* manner.

We now develop a novel and systematic input space division approach that will work for arbitrary shape of input fuzzy sets. Without loss of generality, we still use the above fuzzy controller. Fig. 3 shows the input space division result specifically for the controller with the fuzzy sets shown in Fig. 2. There are four different input space divisions for the four rules, one for each [Fig. 3(a)–(d)]. For clearer illustration and without losing generality, we show the results in $[-15,\ 15] \times [-12,\ 12]$ only, which is a part of the whole input space. Note that there are two regions for each input space division. We called each region an input combination (IC) [19], [22]. For Rule 1, the regions are labeled as A_1 and A_2, and the boundary between them is labeled as boundary A. Similar notations are adopted for Rules 2–4. The corresponding result of the fuzzy AND operation for each region is also given in Fig. 3, side by side with its IC number. For instance, the result for region A_2 is $\mu_{R_{j+1}}$ [Fig. 3(a)].

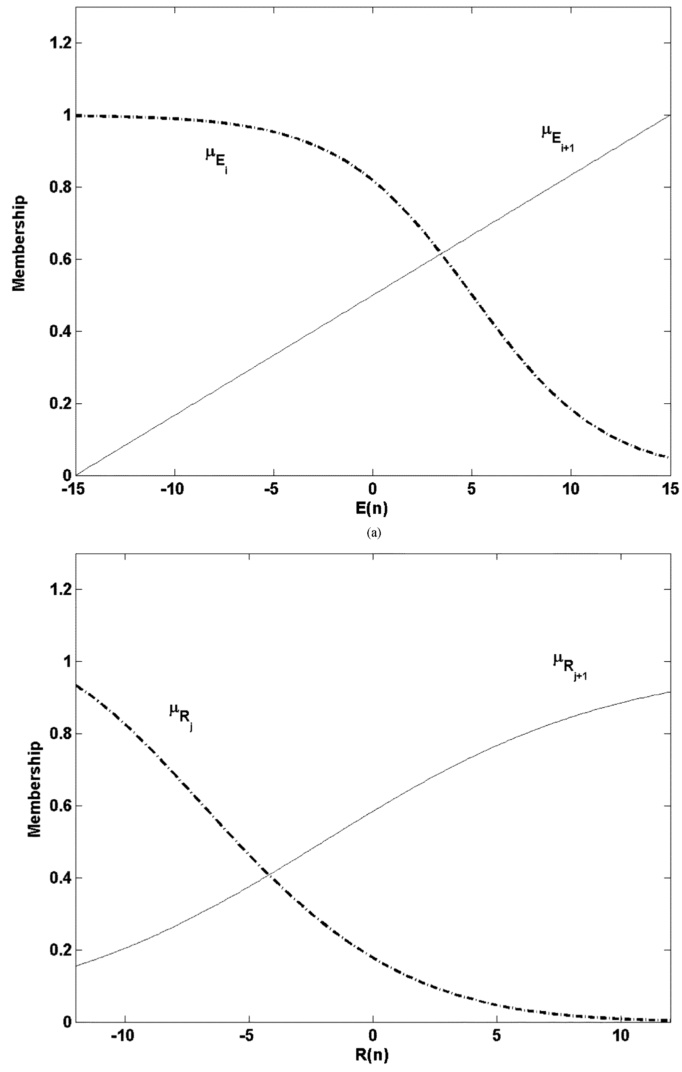

Fig. 2. Two example fuzzy sets for $E(n)$ (a) for μ_{E_i} and $\mu_{E_{i+1}}$. Two example fuzzy sets for $R(n)$ (b) for μ_{R_j} and $\mu_{R_{j+1}}$. They are used to illustrate the process of deriving the mathematical input–output relation of the fuzzy controllers configured in this paper.

We now provide more detail information on how to generate these figures in a systematic manner. Take Fig. 3(a) as an example. Boundary A, on which the membership value is the same between $\mu_{E_{i+1}}$ and $\mu_{R_{j+1}}$, is obtained by letting them equal. That is

$$\frac{E(n) + 15}{30} = \frac{1}{1 + e^{-0.17(R(n)+2)}}$$
$$\text{or } R(n) = -2 + 5.882 Ln \frac{15 + E(n)}{15 - E(n)}.$$

Once the boundary is available, it is trivial to determine which function belongs to which IC—just compute the values of $\mu_{E_{i+1}}$ and $\mu_{R_{j+1}}$ using one pair of $E(n)$ and $R(n)$ in either IC. The function producing the smaller value will be the result for Zadeh AND operation for that IC.

By letting $R(n) = -12$ and $R(n) = 12$ in the last equation, one can calculate the corresponding values for $E(n)$ to be -10.37 and 12.46, respectively [Fig. 3(a)]. We have labeled all

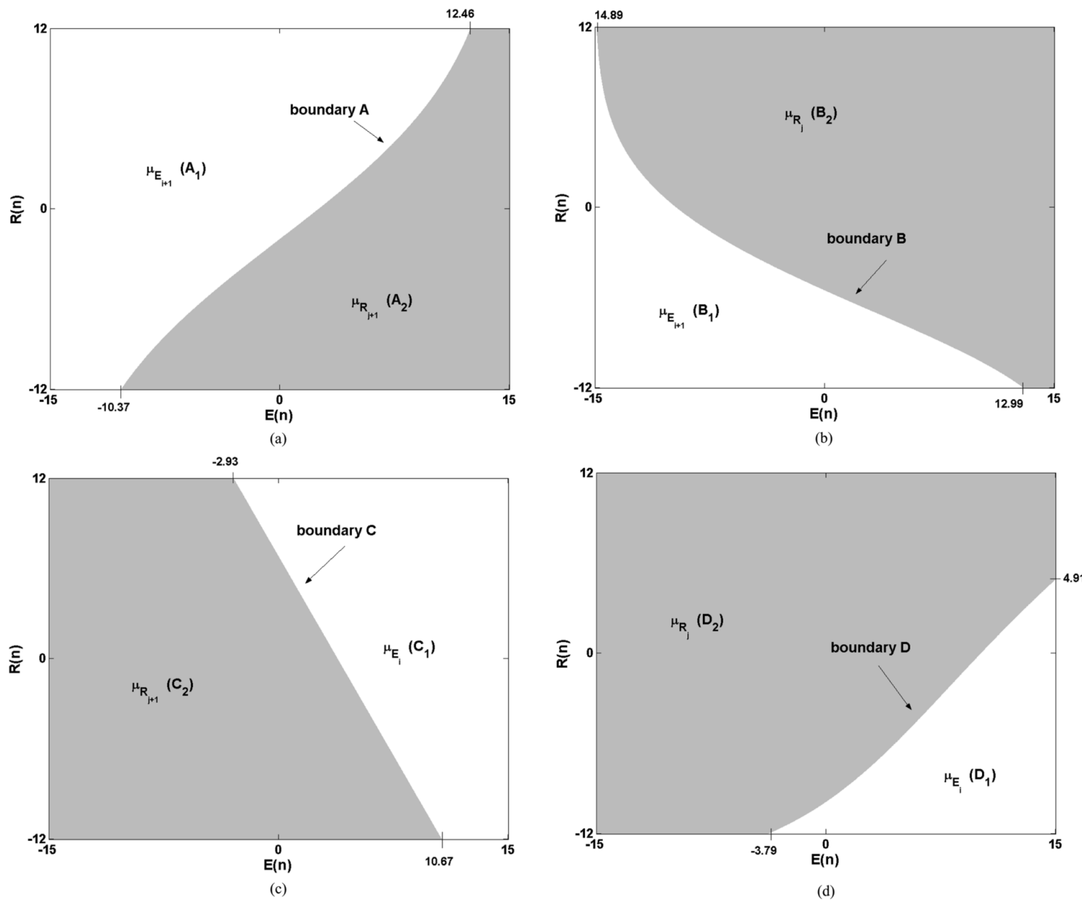

Fig. 3. Input space divisions for each of the four rules of the fuzzy controller with the input fuzzy sets shown in Fig. 2. Fig. 3(a)–(d) correspond to Rules 1–4, respectively.

such intersection points for Fig. 3(b)–(d). For convenience, we also list the other three boundary formulas as follows:

$$\text{boundary B}: \quad R(n) = -15 + 11.4018 \times \sqrt{3.4012 - Ln(E(n)+15)}$$

$$\text{boundary C}: \quad R(n) = -1.765E(n) + 6.825$$

$$\text{boundary D}: \quad R(n) = -15 + 11.4018 \times \sqrt{Ln(1 + e^{0.3(E(n)-5)})}.$$

The divisions in Fig. 3 are for each of the four fuzzy rules when they are evaluated individually. Nevertheless, all the four rules must be ***simultaneously*** considered to produce the fuzzy controller output as well as to derive the input–output structure. This simultaneous consideration is achieved if we superimpose the four individual input space divisions in Fig. 3 to form an overall input space division for all the rules (Fig. 4). It turns out that there are a total of ten ICs, from IC1 to IC10. Note that the number and shape of these regions depend on the four individual input space divisions shown in Fig. 3. The boundaries of each IC are formed by some or all of the boundaries A–D. Their intersection points can be calculated. For instance, IC7 is bounded by boundaries B–D, while IC5 are formed by all four boundaries whose intersection points are (Fig. 4)

$$\text{Point a is } (-2.7715, -4.1988)$$
$$\text{Point b is } (3.4426, 0.6148)$$

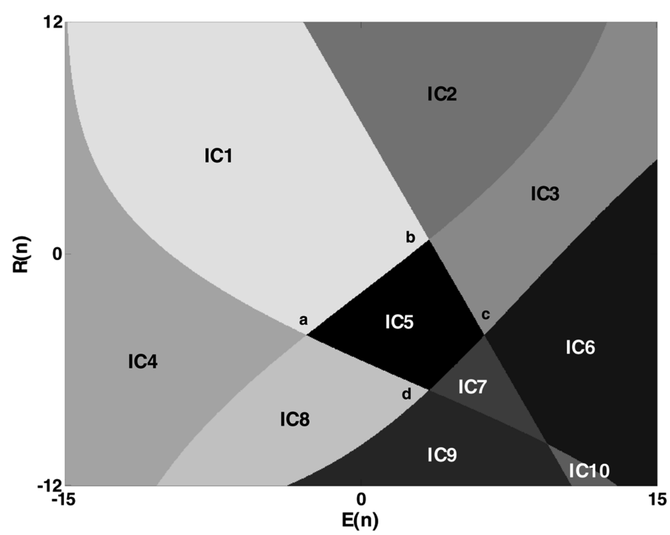

Fig. 4. Overall input space division for all the four rules, which is achieved by superimposing the four individual input space divisions shown in Fig. 3.

$$\text{Point c is } (6.2459, -4.1990)$$
$$\text{Point d is } (3.4423, -7.0469).$$

Now, for each region in Fig. 4, we can decide, using Figs. 3 and 4, the resulting membership function for each fuzzy rule due to Zadeh fuzzy AND operation. For example, IC5 corresponds to A_2 for Rule 1, to B_2 for Rule 2, to C_2 for Rule 3,

TABLE I
RESULT OF ZADEH AND OPERATION IN EACH FUZZY RULE FOR THE OVERALL INPUT SPACE DIVISION SHOWN IN FIG. 4

IC No.	Rule 1	Rule 2	Rule 3	Rule 4
1	$\mu_{\tilde{E}_{i+1}}$ (A_1)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{R}_j}$ (D_2)
2	$\mu_{\tilde{E}_{i+1}}$ (A_1)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{E}_i}$ (C_1)	$\mu_{\tilde{R}_j}$ (D_2)
3	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{E}_i}$ (C_1)	$\mu_{\tilde{R}_j}$ (D_2)
4	$\mu_{\tilde{E}_{i+1}}$ (A_1)	$\mu_{\tilde{E}_{i+1}}$ (B_1)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{R}_j}$ (D_2)
5	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{R}_j}$ (D_2)
6	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{E}_i}$ (C_1)	$\mu_{\tilde{E}_i}$ (D_1)
7	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{R}_j}$ (B_2)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{E}_i}$ (D_1)
8	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{E}_{i+1}}$ (B_1)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{R}_j}$ (D_2)
9	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{E}_{i+1}}$ (B_1)	$\mu_{\tilde{R}_{j+1}}$ (C_2)	$\mu_{\tilde{E}_i}$ (D_1)
10	$\mu_{\tilde{R}_{j+1}}$ (A_2)	$\mu_{\tilde{E}_{i+1}}$ (B_1)	$\mu_{\tilde{E}_i}$ (C_1)	$\mu_{\tilde{E}_i}$ (D_1)

and to D_2 for Rule 4. According to Fig. 3, the membership functions resulted from Zadeh AND operation for these regions are: $\mu_{R_{j+1}}$ for A_2, μ_{R_j} for B_2, $\mu_{R_{j+1}}$ for C_2, and μ_{R_j} for D_2. This process is applied to IC1 to IC10 and the complete result is listed in Table I. Putting these membership functions into the defuzzifier, one gets the analytical structure of the fuzzy controller (Table II). Clearly, it is a discrete-time nonlinear controller with different control algorithms in different ICs.

Like most existing fuzzy controllers, $U(n)$ changes continuously across the boundary of any adjacent ICs (e.g., [19]). On any boundary or at any conjunction point of multiple ICs, $U(n)$ can be computed using any of the control algorithms whose ICs are involved. The result will be the same. It should be noted that the continuity does not necessarily guarantee the existence of the first-order derivative at the boundary, which needs to be studied case by case.

Even though we have used a particular fuzzy controller to demonstrate how the new technique works, it should be obvious that this new method is general and works for any fuzzy controllers that use any type of input fuzzy sets and Zadeh AND operator. Furthermore, it works for two-input fuzzy controllers as well as fuzzy controllers with more than two input variables.

The restrictions applied in Section II-A to the type of fuzzy rules (i.e., Mamdani type instead of TS type), type of output fuzzy sets (i.e., singleton type instead of non-singleton type), type of fuzzy inference, and type of defuzzifier are actually unnecessary and can be removed now (they were needed only for better presentation of the new technique). This is because all the results in the present subsection deal with Zadeh AND operation, which is independent of type of fuzzy rules, type of output fuzzy sets, type of inference, and type of defuzzifier. These components come to play ***after*** Zadeh AND operation is completed.

C. General Relationship Between Shape of Input Fuzzy Sets and Shape of ICS' Boundaries

Due to the use of Zadeh AND operator, the input space is always divided into regions. Different regions have different input–output formulas. Thus, it is important to know how the input space is divided by input fuzzy sets and what they look like in n-dimensional input space. It is not meaningful to study the input–output relationship without studying the regions. We now study how shape of ICs' boundaries is related to shape of input fuzzy sets, and we will do so for both two-input controllers and fuzzy controllers with more than two input variables.

TABLE II
MATHEMATICAL INPUT–OUTPUT RELATION OF THE FUZZY CONTROLLER FOR THE OVERALL INPUT SPACE DIVISION (FIG. 4)

IC No.	$U(n) =$
1	$Ku\dfrac{\frac{1}{30}(15+E(n))h_1+\frac{e^{0.17R(n)}h_3}{0.7118+e^{0.17R(n)}}+e^{-\frac{1}{130}(15+R(n))^2}(h_2+h_4)}{2e^{-\frac{1}{130}(15+R(n))^2}+\frac{1}{1+e^{-0.17(2+R(n))}}+\frac{15+E(n)}{30}}$
2	$Ku\dfrac{\frac{1}{30}(15+E(n))h_1+\frac{4.4817h_3}{4.4817+e^{0.3E(n)}}+e^{-\frac{1}{130}(15+R(n))^2}(h_2+h_4)}{\frac{1}{2}+2e^{-\frac{1}{130}(15+R(n))^2}+\frac{1}{1+0.2231e^{0.3E(n)}}+\frac{E(n)}{30}}$
3	$Ku\dfrac{\frac{h_1}{1+0.7718e^{-0.17R(n)}}+\frac{4.4817h_3}{4.4817+e^{0.3E(n)}}+e^{-\frac{1}{130}(15+R(n))^2}(h_2+h_4)}{2e^{-\frac{1}{130}(15+R(n))^2}+\frac{1}{1+0.2231e^{0.3E(n)}}+\frac{1}{1+e^{-0.17(2+R(n))}}}$
4	$Ku\dfrac{\frac{1}{30}(15+E(n))(h1+h2)+\frac{h3}{1+e^{-0.17(2+R(n))}}+e^{-\frac{1}{130}(15+R(n))^2}h4}{1+e^{-\frac{1}{130}(15+R(n))^2}+\frac{1}{1+0.7118e^{-0.17R(n)}}+\frac{E(n)}{15}}$
5	$Ku\dfrac{0.5(0.7118(h_2+h_4)+e^{0.0077(4.7521+R(n))(47.3479+R(n))}(h_1+h_3)+e^{0.17R(n)}(h_2+h_4))}{0.7118+e^{0.17R(n)}+e^{0.0077(4.7521+R(n))(47.3479+R(n))}}$
6	$Ku\dfrac{\frac{h_1}{1+0.7118e^{-0.17R(n)}}+e^{-\frac{1}{130}(15+R(n))^2}h_2+\frac{4.4817(h_3+h_4)}{4.4817+e^{0.3E(n)}}}{e^{-\frac{1}{130}(15+R(n))^2}+\frac{8.9634}{4.4817+e^{0.3E(n)}}+\frac{1}{1+e^{-0.17(2+R(n))}}}$
7	$Ku\dfrac{e^{-\frac{1}{130}(15+R(n))^2}h_2+\frac{e^{0.17R(n)}(h_1+h_3)}{0.7118+e^{0.17R(n)}}+\frac{4.4817h_4}{4.4817+e^{0.3E(n)}}}{e^{-\frac{1}{130}(15+R(n))^2}+\frac{1}{1+0.2231e^{0.3E(n)}}+\frac{2}{1+e^{-0.17(2+R(n))}}}$
8	$Ku\dfrac{\frac{1}{30}(15+E(n))h_2+\frac{e^{0.17R(n)}(h_1+h_3)}{0.7118+e^{0.17R(n)}}+e^{-\frac{1}{130}(15+R(n))^2}h_4}{e^{-\frac{1}{130}(15+R(n))^2}+\frac{2}{1+e^{-0.17(2+R(n))}}+\frac{15+E(n)}{30}}$
9	$Ku\dfrac{\frac{h_1}{1+e^{-0.17(2+R(n))}}+\frac{1}{30}(15+E(n))h_2+\frac{h_3}{1+e^{-0.17(2+R(n))}}+\frac{4.4817h_4}{4.4817+e^{0.3E(n)}}}{\frac{1}{1+0.2231e^{0.3E(n)}}+\frac{2}{1+e^{-0.17(2+R(n))}}+\frac{15+E(n)}{30}}$
10	$Ku\dfrac{\frac{h_1}{1+e^{-0.17(2+R(n))}}+\frac{1}{30}(15+E(n))h_2+\frac{4.4817h_3}{4.4817+e^{0.3E(n)}}+\frac{4.4817h_4}{4.4817+e^{0.3E(n)}}}{\frac{8.9634}{4.4817+e^{0.3E(n)}}+\frac{1}{1+e^{-0.17(2+R(n))}}+\frac{15+E(n)}{30}}$

In [28], we showed that all the ICs' boundaries of some two-input fuzzy controllers with the trapezoidal input fuzzy sets were formed by straight lines that were related to the trapezoids. This is the case when the fuzzy rules are evaluated individually [similar to Fig. 3(c)] as well as simultaneously (similar to Fig. 5). However, the following significant issue was not studied and had remained an open question before this study had begun.

> Does the above observation from some example controllers actually hold true for ***any*** two-input fuzzy controllers with trapezoidal input fuzzy sets?

Additionally, following are two more general and important issues regarding two-input fuzzy controllers.

1) When fuzzy rules are individually evaluated, can a linear boundary be formed if input fuzzy sets are in shapes other than the trapezoidal ones? If so, what are the conditions for this to happen?

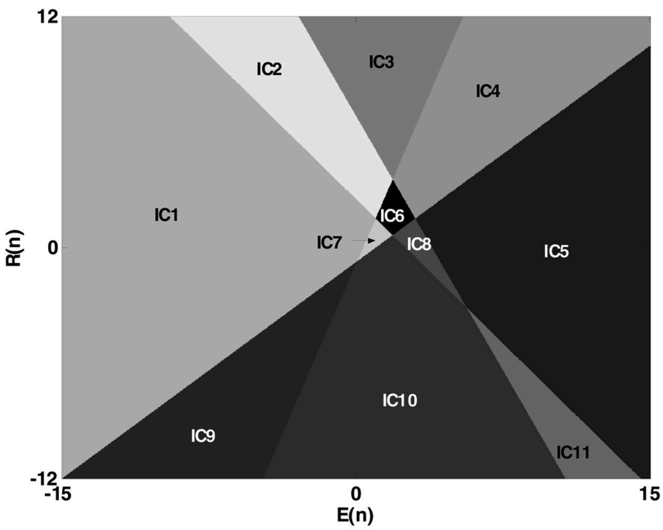

Fig. 5. Overall input space division generated when example μ_{E_i}, $\mu_{R_{E+1}}$, μ_{R_j}, and $\mu_{R_{j+1}}$ have the same type of mathematical function.

2) What are the conditions under which all ICs' boundaries are formed by lines when all the fuzzy rules are evaluated simultaneously?

Boundary C in Fig. 3 indicates that an affirmative answer to the first part of Question 1 is possible. Note that the two membership functions involved to form boundary C are μ_{E_i} and $\mu_{R_{j+1}}$, neither of which is trapezoidal. The boundary is obtained by letting the two membership functions equal

$$\frac{1}{1+e^{0.3(E(n)-5)}} = \frac{1}{1+e^{-0.17(R(n)+2)}}.$$

Simplifying the equation produces the boundary C, a line equation given earlier. The line is resulted because μ_{E_i} and $\mu_{R_{j+1}}$ are of the same type of function, which in this case is the sigmoid function.

We now provide answers to the above three questions below in Theorem 1. As stated earlier already, triangular fuzzy sets are considered as a special case of trapezoidal fuzzy sets. As such, they are considered to be the same type of mathematical function throughout this paper.

Theorem 1: For fuzzy controllers using two input variables and Zadeh AND operator, the boundaries for all ICs are lines if and only if all input fuzzy sets are mathematically of the same type. This is the case when fuzzy rules are evaluated individually or all at once.

Proof: Assume X_1 and X_2 are two (scaled) input variables. Suppose there are total Φ fuzzy rules and the mth rule is

$$\text{IF } X_1 \text{ is } \widetilde{P}_i \text{ AND } X_2 \text{ is } \widetilde{Q}_j \text{ THEN } u(n) \text{ is } \widetilde{H}_k$$

where $\widetilde{H}_k$ is an output fuzzy set. $\widetilde{P}_i$ and $\widetilde{Q}_j$ are two input fuzzy sets of the same mathematical type whose membership functions are $\mu_{P_i} = f(a_1X_1 + b_1)$ and $\mu_{Q_j} = f(a_2X_2 + b_2)$, respectively. Here, f is a convex or monotonic function that is used to indicate that $\widetilde{P}_i$ and $\widetilde{Q}_j$ are of the same type of mathematical functions and a_1, a_2, b_1, and b_2 are design parameters whose values are determined by the controller designer. Note that the boundary for the IC of this rule is formed by letting

$$f(a_1X_1 + b_1) = f(a_2X_2 + b_2)$$

resulting in $a_1X_1+b_1 = a_2X_2+b_2$ or $a_1X_1-a_2X_2+b_1-b_2 = 0$. This is a line on the $X_1 - X_2$ plane. It should be clear that Φ rules will lead to Φ lines when each rule is evaluated individually. This means that all the boundaries are lines when all the rules are evaluated simultaneously. This proves the sufficiency portion of the theorem.

Let us prove the necessity part of the theorem. If at least one input fuzzy set is of a different type than the type of the rest of the fuzzy sets, the boundary whose determination involves that fuzzy set cannot be a line. Without loss of generality, suppose $\widetilde{P}_i$ and $\widetilde{Q}_j$ are now of two different types characterized, respectively, by $\mu_{P_i} = f(a_1X_1 + b_1)$ and $\mu_{Q_j} = g(a_2X_2 + b_2)$, where g is a convex or monotonic function that is different from f. Then, the boundary formed by $f(a_1X_1+b_1) = g(a_2X_2+b_2)$ cannot be a line.

It should be logical to follow from these analyses that the boundaries of all the ICs for any individual fuzzy rule are lines if and only if all the input fuzzy sets related to that rule are mathematically of the same type. Moreover, if and only all the ICs' boundaries are lines when fuzzy rules are evaluated individually, can the boundaries of the ICs be all lines when all the rules are evaluated simultaneously. ■

As a concrete example, let us change $\mu_{E_{i+1}}$ and μ_{R_j} of the fuzzy controller above to the same type of function as μ_{E_i} and $\mu_{R_{j+1}}$

$$\mu_{E_{i+1}} = \frac{1}{1+e^{-0.4(E(n)+0.5)}} \quad -\infty < E(n) < \infty$$
$$\mu_{R_j} = \frac{1}{1+e^{0.4(R(n)-3)}} \quad -\infty < R(n) < \infty.$$

The resulting ICs are shown in Fig. 5. As predicted by Theorem 1, the boundaries of all the ICs are lines.

Although most fuzzy controllers in the literature are of single-input–single-output (SISO) and use two or three input variables, use of more input variables is not impossible and is indeed allowed. Additionally, when a fuzzy controller is used to control more than one physical variable (e.g., temperature and humanity) the controller will be multiple-input, usually requiring at least four input variables (e.g., error and change of error for each physical variable). Thus, there is a need to extend the previous three questions to cover n-input fuzzy controllers. A general question is then as follows.

> When fuzzy rules are evaluated individually or simultaneously for n-input fuzzy controllers using the same type of input fuzzy sets (trapezoidal or any other types), what are the ICs' boundaries and what are the conditions for them to occur?

Let us first study fuzzy controllers with three input variables and then extend the result to n-dimensional cases. One may feel intuitive to say that the boundaries would be arbitrary planes and the ICs would be hexahedrons. Interestingly, this is only partially correct as we show as follows.

Theorem 2: For fuzzy controllers using three input variables and Zadeh AND operator, the boundaries for all ICs are planes in the three-dimensional rectangular coordinate system if and only if all input fuzzy sets are of the same type of mathematical function. Each plane is parallel to one of the three planes: $X_1 - X_2$ plane, $X_1 - X_3$ plane, or $X_2 - X_3$ plane, making all the ICs cuboids or cubes.

Proof: For convenience and brevity, we utilize the material in the proof of Theorem 1 with some modifications to prove the present theorem.

Assume X_3 is the third input variable. Now, the mth rule becomes

$$\text{IF } X_1 \text{ is } \widetilde{P}_i \text{ AND } X_2 \text{ is } \widetilde{Q}_j \text{ AND } X_3 \text{ is } \widetilde{S}_k \text{ THEN } u(n) \text{ is } \widetilde{H}_p$$

where $\widetilde{S}_k$ is an input fuzzy set whose membership is $\mu_{S_k} = f(a_3X_3 + b_3)$. Zadeh AND operator is used

$$\min(\mu_{P_i}, \mu_{Q_j}, \mu_{S_k}) = \min(f(a_1X_1 + b_1), f(a_2X_2 + b_2), f(a_3X_3 + b_3)).$$

Thus, the ICs' boundaries are defined by the following three equations:

$$f(a_1X_1 + b_1) = f(a_2X_2 + b_2)$$
$$f(a_1X_1 + b_1) = f(a_3X_3 + b_3)$$
$$f(a_2X_2 + b_2) = f(a_3X_3 + b_3).$$

They result in the following respective three-dimensional planes in the $X_1 - X_2 - X_3$ coordinate system:

$$a_1X_1 - a_2X_2 + b_1 - b_2 = 0 \quad a_1X_1 - a_3X_3 + b_1 - b_3 = 0$$
$$a_2X_2 - a_3X_3 + b_2 - b_3 = 0.$$

The first plane is parallel to the X_1–X_2 plane, the second to the X_1–X_3 plane, and the third to the X_2–X_3 plane. Because the boundaries formed by all the fuzzy rules are the planes of this nature, all the ICs must be either cuboids or cubes; they cannot be any other shape. This completes the sufficiency proof.

On the other hand, if one of the three fuzzy sets being ANDed, say $\widetilde{Q}_j$, is of a different mathematical function than $\widetilde{P}_i$ and $\widetilde{S}_k$ are (let us denote it g), then planes will not be produced by following two of the three equations in the $\min()$ operations:

$$f(a_1X_1 + b_1) = g(a_2X_2 + b_2)$$
$$g(a_2X_2 + b_2) = f(a_3X_3 + b_3).$$

This proves the necessary condition part of the theorem. ■

With Theorem 2 in mind, the following result is easier to understand and prove.

Theorem 3: For fuzzy controllers using $n(n > 3)$ input variables and Zadeh AND operator, the boundaries for all ICs are hyperplanes in the n-dimensional rectangular system if and only if all input fuzzy sets are of the same type of mathematical function. Each hyperplane is parallel to one of the C_2^n (combinations selecting two out of n) hyperplanes spanned by the following C_2^n axis pairs: $X_1 - X_2, \ldots, X_1 - X_n$, $X_2 - X_3, \ldots, X_2 - X_n, \ldots$, and $X_{n-1} - X_n$. All the ICs are hypercuboids or hypercubes.

Proof: The proof is obvious as it is just an extended version of the proof for Theorem 2. ■

Theorems 1 to 3 hold for any fuzzy controllers that use Zadeh AND operator, regardless of type of fuzzy rules, type of fuzzy inference, and type of defuzzifier. The reason is the same as that given at the end of the last subsection.

III. Implication of the Derived Input–Output Structure for Design of Input Fuzzy Sets

The results obtained in Section II are the first and only ones in the literature. They provide an unprecedented opportunity to rigorously and insightfully examine how different components of a fuzzy controller affect the input–output structure of fuzzy controllers when Zadeh AND operator is involved. In this investigation, we focus on selection of input fuzzy sets because the new structure information has formed a solid basis for studying whether or not any particular type of fuzzy sets, among infinite types in existence, is a sensible choice as input fuzzy sets.

We already know that if the input fuzzy sets are of the trapezoidal type (or the triangular type), the input–output structure of some fuzzy controllers using Zadeh AND operator can be related to conventional control. They are actually nonlinear PI, PD, or PID controllers with variable gains [28] (see also [19] and [25]–[27]). This connection is significant because the PID control dominates 90% industrial process control worldwide [1]. Specifically, according to [28], for a typical two-input fuzzy controller using the trapezoidal input fuzzy sets, in every IC (there are 26 of them), the controller structure can be expressed as $(\Delta U(n) = U(n) - U(n-1))$

$$\Delta U(n) = \frac{C_1e(n) + C_2r(n) + C_3}{D_1e(n) + D_2r(n) + D_3}$$

where C_i and D_i, $i = 1, 2, 3$, are constants. In different ICs, the constants may be different. If we designate

$$K_i(e,r) = \frac{C_1}{D_1e(n) + D_2r(n) + D_3}$$
$$K_p(e,r) = \frac{C_2}{D_1e(n) + D_2r(n) + D_3}$$
$$\delta(e,r) = \frac{C_3}{D_1e(n) + D_2r(n) + D_3}$$

then the algorithm in each of the 26 ICs is a nonlinear PI controller in incremental form with variable proportional-gain $K_p(e,r)$, variable integral-gain $K_i(e,r)$, and variable offset $\delta(e,r)$

$$\Delta U(n) = K_i(e,r)e(n) + K_p(e,r)r(n) + \delta(e,r).$$

The gains and offset vary with change of $e(n)$ and $r(n)$.

Can fuzzy controllers be related to conventional control if other types of input fuzzy sets are used? Let us first look into this issue through the control structure given in Table II as it provides a concrete example. Clearly, this fuzzy controller can be viewed as a discrete-time nonlinear controller developed via the fuzzy control paradigm. This nonlinear controller consists of ten different control algorithms for ten different ICs. This is different from most conventional controllers that mostly have only one control algorithm designed/defined for the entire input space. Moreover, none of the ten algorithms seems to have an understandable structural similarity or connection with conventional controllers. This can lead to failure in understanding, intuitively or otherwise, how and why this fuzzy controller works (if it works). This is the case despite the fact that we already have complete knowledge on the explicit structure of the fuzzy controller.

Unfortunately, this issue is not limited to the fuzzy controllers configured in Section II-A only. Rather, they are general for any (Mamdani) fuzzy controllers that use Zadeh AND operator as long as at least one input fuzzy set is nontrapezoidal. This point becomes clearer if one examines Table II. Some, but not all, of the membership functions $\widetilde{E}_i$, $\widetilde{E}_{i+1}$, $\widetilde{R}_j$, and $\widetilde{R}_{j+1}$ show up in each control algorithm (we purposely do not simplify the algorithms to their simplest mathematical forms so that the involvement of each membership function can be more clearly seen). For instance, in IC1, $\widetilde{E}_{i+1}$, $\widetilde{R}_j$, and $\widetilde{R}_{j+1}$ appear in the numerator and denominator of the control algorithm. Because the membership functions are of different mathematical functions, the algorithm cannot ultimately be simplified or changed to form a control structure recognizable in the context of conventional control (e.g., PID control). The same can be said to the remaining nine algorithms in Table II.

Conventional controllers do not have this problem—one can always clearly understand how and why a controller works based on its algorithm. This problem can also complicate design and analysis of the fuzzy control systems. An issue at point is stability analysis. When the trapezoidal input fuzzy sets are used, we were at least able to develop methods to determine local stability and BIBO stability of the fuzzy control systems through the utilization of the explicit input–output structure of the fuzzy controllers [4], [22], [24]. No global stability result involving the explicit structure exists in the literature even for the fuzzy control systems using the trapezoidal input fuzzy sets. This is partially due to the difficulty of (many) different algorithms in different ICs. One challenge is to construct a Liapunov function for such systems. Global stability conditions established without using the explicit input–output structure knowledge are scarce and are too conservative to be practically useful. It is not clear at this point how to establish local stability, BIBO stability, or global stability for fuzzy control systems using nontrapezoidal input fuzzy sets even when the input–output relation of the fuzzy controllers is known. This appears to be a challenging and interesting research topic.

A fundamental question is then naturally raised: Should fuzzy sets other than the trapezoidal type be used as input fuzzy sets? This is also a practical question since selecting type of input fuzzy sets is a challenging step in fuzzy controller design. There exist many different types of fuzzy sets (an infinite number, to be exact). Little rigorous technical justification has been developed in the literature regarding how to choose input fuzzy sets. Up to date, the selection process is still difficult not only to the novice designer but also sometimes to the expert. Indeed, it is largely a trial-and-error process coupled with some empirical design guidelines established according to personal observation (mostly from knowledge engineering standpoint) and simulation studies. Bounded by lack of in-depth mathematical investigation, the guidelines in the literature can be neither comprehensive nor rigorous. In light of the discussion, we propose the following guideline for input fuzzy set type selection.

Proposition: For any Mamdani fuzzy controller using Zadeh AND operator, the trapezoidal or triangular type should be the first choice most of time for input fuzzy sets. This is because the resulting input–output structure of the fuzzy controller is related to conventional control in an understandable manner, possibly making system analysis and design easier.

We purposely exclude TS fuzzy controllers. Our preliminary study indicates that they represent more complicated circumstances and thus deserve a separate investigation. Also, we have not ruled out the use of other types of fuzzy sets as input fuzzy sets. It remains an interesting open research question with regard to whether or not they have any real advantages over the trapezoidal type and, if so, under which circumstances they should be used. Computer simulation without mathematical support would be too superficial and unconvincing. A rigorous and systematic investigation, nevertheless, is beyond the scope and length of this paper.

By the same token, our Proposition is applicable to automated learning of input fuzzy sets via neural networks [9], [12] or genetic algorithms [7], [29], an active research area in the recent years. According to the literature, there is no restriction on type of input fuzzy sets to be learned by the genetic-algorithm-fuzzy systems. As for neuro-fuzzy systems, input fuzzy sets to be learned must be nontrapezoidal types. This is largely because if the first derivative of an input fuzzy set does not exist at some location in the universe, which is the case for the trapezoidal fuzzy sets, the learning algorithms of the neuro-fuzzy systems will fail to operate [9].

If one merely treats a neuro-fuzzy controller or genetic-algorithm-fuzzy controller as a black-box controller without wanting to know its mathematical input–output relationship and conduct rigorous analysis in the context of conventional control theory, Proposition may be less an issue. However, we need to remind the reader that rigorous analysis and design of the neural network controllers have been achieved (e.g., [8]). The same may be achievable for neuro-fuzzy controllers and genetic-algorithm-fuzzy systems, and if so, should be actively pursed to put the neuro-fuzzy systems and genetic-algorithm-fuzzy systems on a firmer theoretical ground. From this perspective, our Proposition is relevant and can be useful.

IV. Conclusion

We have developed a general technique for rigorously deriving analytical input–output structure for fuzzy controllers that use Zadeh fuzzy AND operator and arbitrary types of input fuzzy sets. We have also established some general necessary and sufficient conditions relating the shape of input fuzzy sets to the shape of input space divisions. The relationships as well as the technique are applicable to any fuzzy controllers that use Zadeh AND operator, regardless of type of fuzzy rules, type of output fuzzy sets (for Mamdani rules) or type of functions in rule consequents (TS rules), type of fuzzy inference, and type of defuzzifier. They are also directly applicable to comparable fuzzy models and fuzzy expert systems.

Furthermore, we have examined the design issue of selecting input fuzzy set type by examining the new input–output structures that we derived. We have found that the fuzzy controllers using the trapezoidal/triangular input fuzzy sets can be understood (and possibly analyzed and designed) most sensibly and easily in the context of control theory. Consequently, we recommend that the trapezoidal/triangular fuzzy sets be used as the first choice if the understanding of the input–output relation of a fuzzy controller is important.

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