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Brief Paper

The Simplest Fuzzy Controllers using Different Inference Methods are Different Nonlinear Proportional-integral Controllers with Variable Gains*

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Key Words—Adaptive control; control system analysis; control theory; fuzzy control; fuzzy systems; nonlinear control systems; PID control; stability.

Abstract—The author analytically proves that the simplest fuzzy controllers using different inference methods are different nonlinear proportional-integral (PI) controllers with proportional-gains and integral-gains changing with inputs of the controllers. The inference methods involved are Mamdani's minimum inference method, Larsen's product inference method, the drastic product inference method and the bounded product inference method. Configuration of the fuzzy controllers is minimal, which includes two input fuzzy sets, three output fuzzy sets, four control rules, Zadeh fuzzy logic AND, Lukasiewicz fuzzy logic OR and a center of gravity defuzzification algorithm. After analytically investigating properties of the nonlinear PI controllers, the author reveals that the bounded product inference method is inappropriate for the control purpose while the other three inference methods are appropriate. Dynamic and static control behaviors of the fuzzy controllers with the appropriate inference methods are analytically compared with each other, and are also compared with those of the linear PI controller. Finally, it is analytically proven that the fuzzy control systems have the same local stability at the equilibrium point as the corresponding linear PI control system does.

1. *Introduction*

ALTHOUGH fuzzy control technology has been used to solve many practical control problems (e.g. Bare *et al.*, 1990; Clymer *et al.*, 1992; Esogbue *et al.*, 1992; Liaw and Wang, 1992; Ying *et al.*, 1992a; Yoshida and Wakabayashi, 1992), solid fuzzy control theory is still in its infancy. A novel method was first developed (Ying, 1987) to analytically analyze the structure of the simplest fuzzy controller. The simplest fuzzy controller that employed the drastic product inference method was theoretically proven to be a nonlinear PI controller (Ying *et al.*, 1990), which was later proven to be the most nonlinear fuzzy controller (Ying, 1993).

A fuzzy controller with strong nonlinearity is generally of practical importance and theoretical interest. Therefore, the primary objective of this research is to continue to reveal analytical structures of the simplest fuzzy controllers using inference methods other than the drastic product inference method, namely Mamdani's minimum inference method (R_M), Larsen's product inference method (R_L) and the bounded product inference method (R_{BP}) (Mizumoto, 1988). These analytical structures can serve as a platform on which analyses can be *theoretically* and *mathematically* developed. Taking advantage of the available analytical structures of the simplest fuzzy controllers, the author will first carry out mathematical analyses to determine the adequacy of the inference methods in the context of fuzzy control, then theoretically compare dynamic and static (including local stability at the equilibrium point) characteristics of the fuzzy controllers that use the appropriate inference methods. Finally, the author will theoretically compare the behavior of the fuzzy controllers with that of the linear PI controller. Because the fuzzy controllers in this study are actually nonlinear adaptive PI controllers, the relationship between fuzzy controllers and adaptive controllers will be discussed.

The four tasks listed above have not yet been tackled in a rigorous mathematical manner. However, it should be pointed out that a comparison study on various fuzzy reference methods in the context of fuzzy control has been conducted by Mizumoto (1988) using computer simulation. Mizumoto employed computer simulation to compare 12 inference methods for fuzzy control, using a fuzzy controller with a specific configuration and a first-order with a time delay process model. The configuration was different from that of the simplest fuzzy controllers studied in this paper. The inference methods included R_M, R_L, R_{DP}, R_{BP}, the standard sequence inference method, the Gödelian logic method and the Gougen method. Based on the simulation study, it was concluded that the fuzzy controllers using R_M, R_L, R_{DP} and R_{BP} produced much better control results than the fuzzy controllers using the rest of eight inference methods did. The research was important because it provided initial valuable comparison results. Also, the results provided guidelines for selecting the inference methods which *might* be appropriate for fuzzy control.

Nevertheless, because the comparison was based on computer simulation and one single process model, the research was a case study and consequently the amount of information one could derive from the results was rather limited. Further, caution should be exercised when interpreting the results because different conclusions might result when different process models are involved, especially when nonlinear process models are involved. Any simulation study, including Mizumoto's simulation study, is constrained by drawbacks of simulation. Although the simulation results demonstrated that R_M, R_L, R_{DP} and R_{BP} were much better than the other inference methods, the following two crucial questions remain unanswered. The first question is: are R_M, R_L, R_{DP} and R_{BP} even valid inference methods in the context of control, despite the fact that they produced reasonably good control results in a single model simulation

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study? The second question is: among valid inference methods (assume the inference methods can be validated), which inference method is the best? It is obvious that questions like these cannot be answered merely using computer simulation. Only through rigorous mathematical analysis can they be satisfactorily addressed.

In this paper, the author will provide theoretical answers to these two questions, among others, in the process of accomplishing the above-specified four important tasks of this research. The research will deal with the fuzzy controllers themselves without any involvement of computer simulation and specific process models. This is because when other components are fixed, it is the inference methods, not process models, that determine the ultimate structures of the fuzzy controllers. In that sense, the involvement of any specific process models is undesirable and consequently should be avoided whenever possible. Simulation study using process models is justifiable only when analytical analysis means are unavailable.

The research will focus on the fuzzy controllers using R_M, R_L, R_{DP} and R_{BP}. The other eight inference methods are excluded in the study. According to the definitions of the standard sequence inference method, the Gödelian logic method and the Gougen method, these inference methods obviously produce positive feedback instead of desirable negative feedback. The other five inference methods seem to be unusable for control purposes because they produced far worse control results than R_M, R_L, R_{DP} and R_{BP} in Mizumoto's simulation study.

2. *Analytical analysis of structure of the simplest fuzzy controllers using the different inference methods*

2.1. *Components of the simplest fuzzy controllers.* The fuzzy controllers studied in this paper are configured by the following components. Because the configuration is minimal for any usable fuzzy controllers, the controllers are called simplest fuzzy controllers.

Inputs of the fuzzy controllers are error and rate change of error (rate, for short) of process output. After scaled, the inputs become

$$GE \cdot e(nT) = GE(y(nT) - \text{setpoint}) \quad (2.1)$$

$$GR \cdot r(nT) = GR(e(nT) - e(nT - T)), \quad (2.2)$$

where GE and GR are scalars for the error and rate, respectively. $y(nT)$ is the process output at sampling time nT and $e(nT - T)$ is error at previous sampling time. The setpoint is a target value for the process output.

The scaled error and rate are fuzzified respectively by input fuzzy sets whose membership functions are shown in Fig. 1. Inside the interval $[-L, L]$, the memberships increase or decrease linearly with the scaled error and rate. Outside $[-L, L]$, the memberships are either 0 or 1. The input fuzzy sets for the scaled error are error.positive and error.negative whose respective membership functions are

$$\mu_e^+ = \frac{L + GE \cdot e(nT)}{2L} \quad \text{and} \quad \mu_e^- = \frac{L - GE \cdot e(nT)}{2L}. \quad (2.3)$$

The input fuzzy sets for the scaled rate are rate.positive and rate.negative whose membership functions can be expressed respectively as

$$\mu_r^+ = \frac{L + GR \cdot r(nT)}{2L} \quad \text{and} \quad \mu_r^- = \frac{L - GR \cdot r(nT)}{2L}. \quad (2.4)$$

It is obvious that

$$\mu_e^+ + \mu_e^- = 1 \quad \text{and} \quad \mu_r^+ + \mu_r^- = 1. \quad (2.5)$$

Four fuzzy control rules are used, namely

IF $GE \cdot e(nT)$ is error.positive
AND $GR \cdot r(nT)$ is rate.positive
THEN $\Delta u(nT)$ is output.negative (r_1)

IF $GE \cdot e(nT)$ is error.positive
AND $GR \cdot r(nT)$ is rate.negative
THEN $\Delta u(nT)$ is output.zero (r_2)

IF $GE \cdot e(nT)$ is error.negative
AND $GR \cdot r(nT)$ is rate.positive
THEN $\Delta u(nT)$ is output.zero (r_3)

IF $GE \cdot e(nT)$ is error.negative
AND $GR \cdot r(nT)$ is rate.negative
THEN $\Delta u(nT)$ is output.positive. (r_4)

In the control rules, $\Delta u(nT)$ is crisp incremental output of the fuzzy controllers. Three trapezoidal-shaped incremental output fuzzy sets, output.positive, output.zero and output.negative, as shown in Fig. 2, are employed. In the figure, $2A$ and $2H$ are the upper and lower sides of the trapezoids, respectively. Also, H is the central value of output.positive and $-H$ is the central value of output.negative. To define shape of the trapezoids, a parameter

$$\theta = \frac{A}{H} \quad (2.6)$$

is employed, which is constrained by

$$\theta \le 0.5 \quad (2.7)$$

to avoid overlay between upper-sides of two adjacent output fuzzy sets.

Four different inference methods are employed. Formal definitions of the methods are given in Table 1 and are illustrated graphically in Fig. 3. In the table, μ is a membership of an output fuzzy set, which is calculated from the input fuzzy sets in a control rule using Zadeh fuzzy logic AND (Min operator). The shadowed areas in Fig. 3 represent the results of the inferences and can be computed according to the formulas in Table 2. It should be pointed out that in this paper superscripts or subscripts M, L, DP and BP always denote respective inference methods.

The control rules r_2 and r_3 generate two memberships,

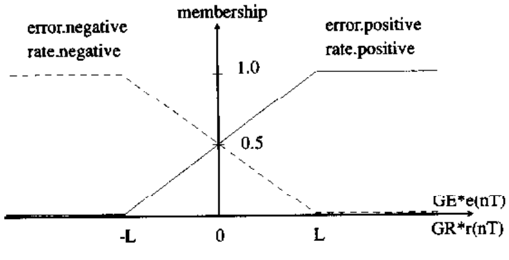


FIG. 1. Illustrative definitions of input fuzzy sets, error.positive, error.negative, rate.positive and rate.negative, for fuzzifying the scaled inputs.

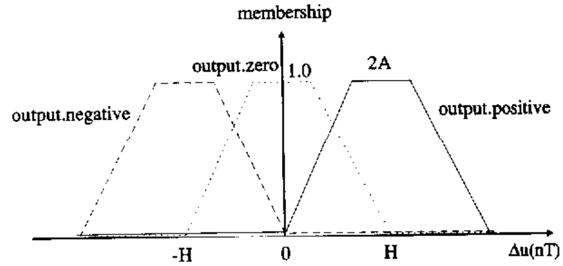


FIG. 2. Illustrative definitions of output fuzzy sets, output.positive, output.zero and output.negative, which have central values H, 0 and $-H$, respectively. Note that $2A$ and $2H$ are the upper and lower sides of the trapezoids.

TABLE 1. THE FORMAL DEFINITIONS OF MAMDANI'S MINIMUM INFERENCE METHOD (R_M), LARSEN'S PRODUCT INFERENCE METHOD (R_L), THE DRASTIC PRODUCT INFERENCE METHOD (R_{DP}) AND THE BOUNDED PRODUCT INFERENCE METHOD (R_{BP})

Inference method	Definition
R_M	$\mu \wedge F(\Delta u)$
R_L	$\mu \cdot F(\Delta u)$
R_{DP}	$\begin{cases} \mu, & F(\Delta u) = 1 \\ F(\Delta u), & \mu = 1 \\ 0, & \mu < 1 \text{ and } F(\Delta u) < 1 \end{cases}$
R_{BP}	$0 \vee [\mu + F(\Delta u) - 1]$

μ is the membership obtained by evaluating the input fuzzy sets in the control rules using Zadeh fuzzy logic AND.
$F(\Delta u)$ is the membership function of the output fuzzy set.
The definitions are also illustratively shown in Fig. 3.

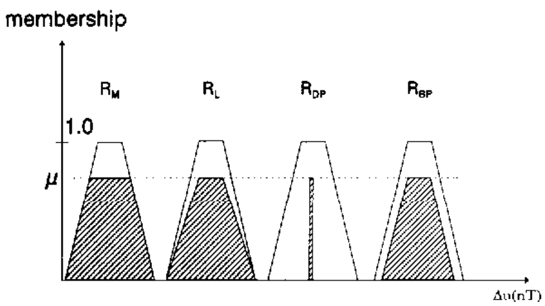


FIG. 3. Illustrative definitions of four inference methods. (a) Mamdani's minimum inference method, denoted as R_M; (b) Larsen's product inference method, denoted as R_L; (c) the drastic product inference method, denoted as R_{DP}; and (d) the bounded product inference method, denoted as R_{BP}. In the definitions, μ is a resultant membership produced by using Zadeh fuzzy logic AND (Min operator) on the input fuzzy sets in a control rule. The shadowed areas are the results of the inferences, which can be calculated according to the formulas in Table 2. The membership functions of the output fuzzy sets are designated as $F(\Delta u)$.

TABLE 2. THE AREA COMPUTATION FORMULAE OF THE OUTPUT FUZZY SETS FOR FOUR DIFFERENT INFERENCE METHODS DEFINED IN TABLE 1

Inference method	Area of the output fuzzy sets
R_M	$S_M(\mu) = \mu(2 - \mu + \mu \cdot \theta)H$
R_L	$S_L(\mu) = \mu(1 + \theta)H$
R_{DP}	$S_{DP}(\mu) = 2\mu \cdot \theta \cdot H$
R_{BP}	$S_{BP}(\mu) = \mu(2\theta + \mu - \mu \cdot \theta)H$

The trapezoidal-shaped output fuzzy sets are defined in Fig. 2.

namely μ_{r_2} and μ_{r_3}, which are maximally negatively correlated for the same output fuzzy set, output.zero (Ying *et al.*, 1988). Consequently, Lukasiewicz fuzzy logic OR should be used to attain a combined membership

$$\mu_{r_2 \vee r_3} = \mathrm{Min}\,(\mu_{r_2} + \mu_{r_3}, 1). \tag{2.8}$$

The popular center of gravity algorithm is used to defuzzify the output fuzzy sets. Because the output fuzzy sets are symmetrical about their central values (H, 0 and $-H$), the global centroid can be calculated from local centroids. The local centroids are always H for output.positive, 0 for output.zero and $-H$ for output.negative. Importance of each local centroid in the global centroid is weighted by the size of area of the respective output fuzzy set. Thus, scaled $\Delta u(nT)$ is computed as

$$\begin{aligned} GU \cdot \Delta u(nT) &= GU \times \frac{-H \cdot S(\mu_{r_1}) + 0 \cdot S(\mu_{r_2 \vee r_3}) + H \cdot S(\mu_{r_4})}{S(\mu_{r_1}) + S(\mu_{r_2 \vee r_3}) + S(\mu_{r_4})} \\ &= GU \cdot H \frac{S(\mu_{r_4}) - S(\mu_{r_1})}{S(\mu_{r_1}) + S(\mu_{r_2 \vee r_3}) + S(\mu_{r_4})} \end{aligned} \tag{2.9}$$

where $S(\mu_{r_1})$ and $S(\mu_{r_4})$, calculated according to the formulae in Table 2, are the areas of output.negative and output.positive with the respective memberships, μ_{r_1} and μ_{r_4}, which are generated by the control rules r_1 and r_4. $S(\mu_{r_2 \vee r_3})$ is the area of output.zero generated by the control rules r_2 and r_3, with the combined membership $\mu_{r_2 \vee r_3}$ calculated according to (2.8). GU is a scalar for $\Delta u(nT)$. The new output of the fuzzy controllers is

$$u(nT) = u(nT - T) + GU \cdot \Delta u(nT) \tag{2.10}$$

where $u(nT - T)$ is output of the fuzzy controllers at sampling time $(n - 1)T$.

2.2. *Analytical structures of the simplest fuzzy controllers using the different inference methods.* In practice, the scaled error and rate of a fuzzy control system are usually managed, in one way or the other, to stay inside the interval $[-L, L]$ in order to take full advantage of nonlinearity of the fuzzy controller. Therefore, in the following sections, properties of the fuzzy controllers when the scaled inputs are inside the interval $[-L, L]$ will be analytically analyzed.

Theorem 1. The simplest fuzzy controllers using R_M, R_L, R_{DP} and R_{BP} are different nonlinear PI controllers with variable gains

$$GU \cdot \Delta u(nT) = -(K_i \cdot e(nT) + K_p \cdot r(nT)) \tag{2.11}$$

where proportional-gain and integral-gain are

$$K_p = \beta \cdot GR \tag{2.12}$$

$$K_i = \beta \cdot GE. \tag{2.13}$$

The βs for different inference methods are listed in Table 3.

Proof. In the interest of brevity, the author will only prove the structure of the fuzzy controller using R_M. The structures of the other fuzzy controllers can be proven in the same way.

Figure 4 shows graphically four possible input combinations (IC) of the scaled error and rate. In each IC, the memberships of the output fuzzy sets in the control rules r_1 to r_4 can be obtained as those listed in Table 4, using Zadeh fuzzy logic AND. For the combined membership of output.zero, it can be easily proven that

$$\mu_{r_2 \vee r_3} = \mathrm{Min}(\mu_{r_2} + \mu_{r_3}, 1) = \mu_{r_2} + \mu_{r_3} \le 1. \tag{2.14}$$

Replacing μ in $S_M(\mu)$ in Table 2 by μ_{r_i} ($i = 1, 2, 3, 4$) in Table 4 and substituting the resulting $S_M(\mu_{r_i})$ into the defuzzification algorithm (2.9), the analytical structure of the fuzzy controller using R_M can be obtained as follows:

when $GR \cdot |r(nT)| \le GE \cdot |e(nT)| \le L$ (the IC1 and IC3 regions)

$$GU \cdot \Delta u(nT) = \frac{0.5H \cdot GU[(1+\theta)L(GE \cdot e(nT) + GR \cdot r(nT)) + 0.5(1-\theta)((GE \cdot e(nT))^2 - (GR \cdot r(nT))^2)]}{(3+\theta)L^2 - [(1+\theta)L \cdot GE\,|e(nT)| + 0.5(1-\theta)((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2)]} \tag{2.15}$$

when $GE \cdot |e(nT)| \le GR \cdot |r(nT)| \le L$ (the IC2 and IC4

TABLE 3. THE EXPRESSIONS OF βs FOR THE DIFFERENT INFERENCE METHODS WHEN BOTH THE SCALED ERROR AND RATE ARE WITHIN THE INTERVAL $[-L, L]$

Inference method	β
R_M	$\beta^M = \dfrac{0.5H \cdot GU[(1+\theta)L + 0.5(1-\theta)\,\lvert GE \cdot e(nT) - GR \cdot r(nT)\rvert]}{(3+\theta)L^2 - [(1+\theta)L \cdot \text{input} + 0.5(1-\theta)((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2)]}$
R_L	$\beta^L = \dfrac{0.5H \cdot GU}{2L - \cdot \text{input}}$
R_{DP}	$\beta^{DP} = \dfrac{0.5H \cdot GU}{2L - \cdot \text{input}}$
R_{BP}	$\beta^{BP} = \dfrac{0.5H \cdot GU[(1+\theta)L - 0.5(1-\theta)\,\lvert GE \cdot e(nT) - GR \cdot r(nT)\rvert]}{(1+3\theta)L^2 - [(1+\theta)L \cdot \text{input} - 0.5(1-\theta)((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2]}$

$$\text{Input} = \begin{cases} GE \cdot \lvert e(nT)\rvert, & \text{IC1 and IC3} \\ GR \cdot \lvert r(nT)\rvert, & \text{IC2 and IC4.} \end{cases}$$

Note that the proportional-gains and integral-gains of the nonlinear PI controllers are $K_p = GR \cdot \beta$ and $K_i = GR \cdot \beta$, respectively. GE and GR are the scalars for the error and rate of the process output. The inference methods are defined in Table 1 and the input combinations IC1 to IC4 are shown graphically in Fig. 4.

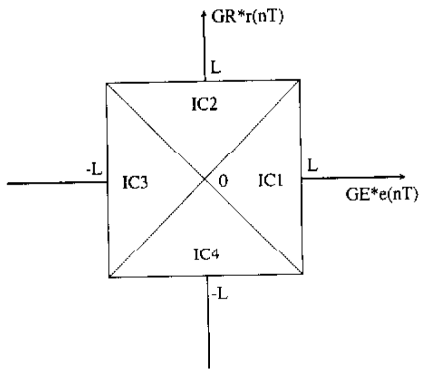


FIG. 4. Four possible input combinations (IC) of the scaled error, $GE \cdot e(nT)$, and scaled rate, $GR \cdot \gamma(nT)$ when both of them are within the interval $[-L, L]$.

regions)

$$GU \cdot \Delta u(nT) = \frac{\begin{array}{l}0.5H \cdot GU[(1+\theta)L(GE \cdot e(nT) + GR \cdot r(nT)) \\ \quad -0.5(1-\theta)((GE \cdot e(nT))^2 - (GR \cdot r(nT))^2)]\end{array}}{\begin{array}{l}(3+\theta)L^2 - [(1+\theta)L \cdot GR\,\lvert r(nT)\rvert \\ \quad +0.5(1-\theta)((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2)]\end{array}}. \tag{2.16}$$

Note that a linear discrete-time PI controller in incremental form is

$$\Delta u_{PI}(nT) = -(K_i^{PI} \cdot e(nT) + K_p^{PI} \cdot r(nT)) \tag{2.17}$$

The equations (2.15) and (2.16) can be concisely expressed as a nonlinear PI controller

$$GU \cdot \Delta u(nT) = -(K_i^M \cdot e(nT) + K_p^M \cdot r(nT)) \tag{2.18}$$

where the variable proportional-gain and integral-gain are

$$K_p^M = \beta^M \cdot GR \tag{2.19}$$

$$K_i^M = \beta^M \cdot GE \tag{2.20}$$

and

$$\beta^M = \begin{cases} \dfrac{\begin{array}{l}0.5H \cdot GU[(1+\theta)L + 0.5(1-\theta) \\ \qquad \times \lvert GE \cdot e(nT) - GR \cdot r(nT)\rvert]\end{array}}{\begin{array}{l}(3+\theta)L^2 - [(1+\theta)L \cdot GE\,\lvert e(nT)\rvert + 0.5(1-\theta) \\ \qquad \times ((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2)]\end{array}}, & \text{IC1 and IC3} \\[2ex] \dfrac{\begin{array}{l}0.5H \cdot GU[(1+\theta)L + 0.5(1-\theta) \\ \qquad \times \lvert GE \cdot e(nT) - GR \cdot r(nT)\rvert]\end{array}}{\begin{array}{l}(3+\theta)L^2 - [(1+\theta)L \cdot GR\,\lvert r(nT)\rvert + 0.5(1-\theta) \\ \qquad \times ((GE \cdot e(nT))^2 + (GR \cdot r(nT))^2)]\end{array}}, & \text{IC2 and IC4.} \end{cases} \tag{2.21}$$ ■

According to Table 3, there obviously exists a structural duality between $GE \cdot e(nT)$ in the IC1 and IC3 regions and $GR \cdot r(nT)$ in the IC2 and IC4 regions.

TABLE 4. THE RESULTS OF EVALUATING THE MEMBERSHIPS OF THE OUTPUT FUZZY SETS IN THE CONTROL RULES r_1 TO r_4 USING ZADEH FUZZY LOGIC AND WHEN BOTH THE SCALED ERROR AND RATE ARE WITHIN THE INTERVAL $[-L, L]$

Input combinations (IC) of scaled error and rate	Memberships of the output fuzzy sets obtained by evaluating the input fuzzy sets in the control rules r_1–r_4 using Zadeh fuzzy logic AND			
	μ_{r_1}	μ_{r_2}	μ_{r_3}	μ_{r_4}
IC1	μ_r^+	μ_r^-	μ_e^-	μ_e^-
IC2	μ_e^+	μ_r^-	μ_e^-	μ_r^-
IC3	μ_e^+	μ_e^+	μ_r^+	μ_r^-
IC4	μ_r^+	μ_e^+	μ_r^+	μ_e^-

The input combinations IC1 to IC4 are shown graphically in Fig. 4.

3. *Properties of the simplest fuzzy controllers using the different inference methods*

3.1. *Properties of the variable proportional-gains and integral-gains.* When GE and GR are fixed, the proportional-gains and integral-gains in (2.12) and (2.13) change proportionally as βs change. Therefore, studying properties of βs for the different inference methods is equivalent to studying properties of the gains.

For all four inference methods, βs have the following properties

$$\beta(e(nT), r(nT)) = \beta(r(nT), e(nT)) \tag{3.1}$$

$$\beta(e(nT), r(nT)) = \beta(-r(nT), -e(nT)). \tag{3.2}$$

The equation (3.1) indicates that βs are symmetrical about the line

$$GR \cdot r(nT) = GE \cdot e(nT), \tag{3.3}$$

while the equation (3.2) states that βs are symmetrical about the line

$$GR \cdot r(nT) = -GE \cdot e(nT). \tag{3.4}$$

Obviously, it is sufficient to study properties of βs in one of the IC1 to IC4 regions. Without losing generality, properties of βs in the IC1 region will be investigated.

3.1.1. *Properties of* β^M. β^M reaches its maximum when

$$GE \cdot e(nT) = L \quad \text{and} \quad GR \cdot r(nT) = -L \tag{3.5}$$

because the numerator of β^M in Table 3 becomes maximal while the denominator becomes minimal. The maximal β^M is

$$\beta^M_{max} = \frac{H \cdot GU}{(1+\theta)L}. \tag{3.6}$$

β^M attains its minimum when

$$e(nT) = r(nT) = 0 \tag{3.7}$$

because the numerator of β^M becomes minimal while the denominator becomes maximal. The minimal β^M is

$$\beta^M_{Min} = \frac{(1+\theta)H \cdot GU}{2(3+\theta)L}. \tag{3.8}$$

Ratio of β^M_{max} to β^M_{min} is

$$\rho^M = \frac{\beta^M_{max}}{\beta^M_{min}} = \frac{2(3+\theta)}{(1+\theta)^2} \tag{3.9}$$

which strictly monotonically decreases as θ increases from 0 to 0.5. Range of ρ^M is

$$\frac{28}{9} \le \rho^M \le 6 \tag{3.10}$$

with the maximal ρ^M occurring when $\theta = 0$.

In the IC1 region, $GE \cdot e(nT) \ge GR \cdot r(nT)$. For a given $GR \cdot r(nT)$, increase of $GE \cdot e(nT)$ makes the numerator of β^M larger and the denominator smaller, causing β^M to increase. For a given $GE \cdot e(nT)$, increase of $GR \cdot r(nT)$ also causes β^M to increase. When $GE \cdot e(nT) = L$ and $GR \cdot r(nT) = L$, β^M becomes

$$\beta^M_{L,L} = \frac{H \cdot GU}{2L}. \tag{3.11}$$

On the basis of the properties of β^M in the IC1 region and the symmetry described in (3.1) and (3.2), it is concluded that starting from the minimum at $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$, β^M strictly monotonically increases with increase of $GE \cdot |e(nT)|$ and $GR \cdot |r(nT)|$ in all directions. To visually confirm the properties analyzed above, a three-dimensional plot of β^M for $\theta = 0$ is demonstrated in Fig. 5.

3.1.2. *Properties of* β^L *and* β^{DP}. When R_L and R_{DP} are used, $\beta^L = \beta^{DP}$ according to Table 3, meaning that the structures of the fuzzy controllers are identical. It is obvious that when

$$GE \cdot e(nT) = L \quad \text{or} \quad GR \cdot |r(nT)| = L, \tag{3.12}$$

β^L and β^{DP} reach their maximum

$$\beta^L_{max} = \beta^{DP}_{max} = \frac{H \cdot GU}{2L} \tag{3.13}$$

and when

$$GT \cdot e(nT) = GR \cdot r(nT) = 0, \tag{3.14}$$

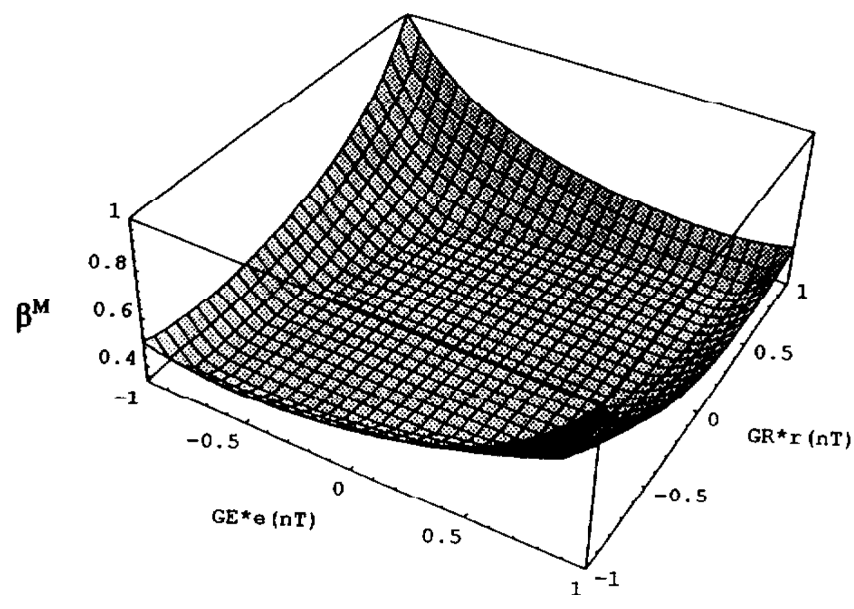


FIG. 5. A three-dimensional plot of β^M for visualizing the properties of β^M analyzed in the text. For the plot, $\theta = 0$, $L = 1$, $GE = 1$, $GR = 1$, $H = 1$ and $GU = 1$. It can be seen that starting from the minimum at $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$, β^M strictly monotonically increases with increasing $GE \cdot |e(nT)|$ and $GR \cdot |r(nT)|$ in all directions. β^M achieves its maximum when $(GE \cdot e(nT), GR \cdot r(nT)) = (L, -L)$ or $(GE \cdot e(nT), GR \cdot r(nT)) = (-L, L)$.

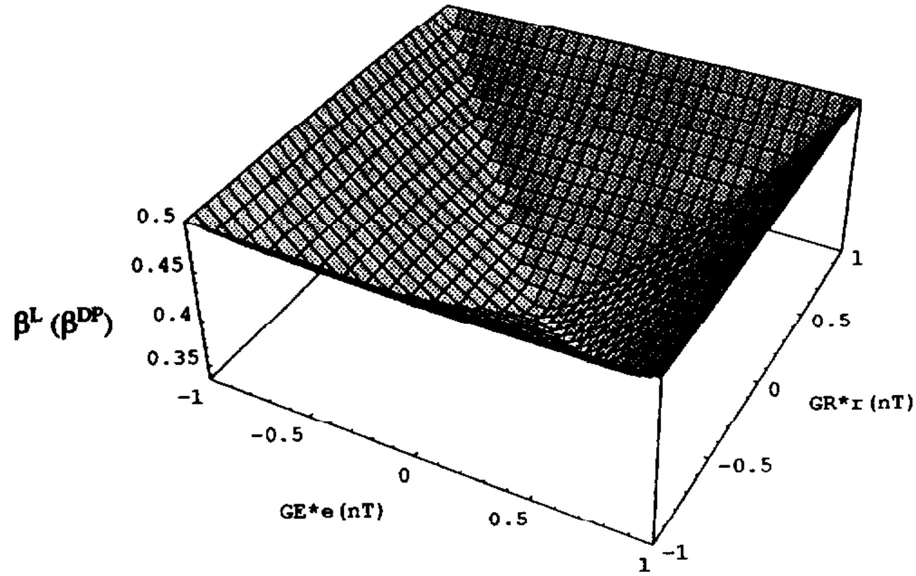


FIG. 6. A three-dimensional plot of β^L and β^{DP} for visualizing the properties of β^L and β^{DP} analyzed in the text. For the plot, $\theta = 0$, $L = 1$, $GE = 1$, $GR = 1$, $H = 1$ and $GU = 1$. It can be seen that starting from the minimum at $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$, β^L and β^{DP} strictly monotonically increase with increase of $GE \cdot |e(nT)|$ and $GR \cdot |r(nT)|$ in all directions. β^L and β^{DP} achieve their maximum when $GE \cdot |e(nT)| = L$ or $GR \cdot |r(nT)| = L$.

β^L and β^{DP} attain their minimum

$$\beta^L_{min} = \beta^{DP}_{min} = \frac{H \cdot GU}{4L}. \quad (3.15)$$

Note that β^L and β^{DP} are independent of θ and therefore

$$\rho^L = \rho^{DP} = \frac{\beta^L_{max}}{\beta^L_{min}} = \frac{\beta^{DP}_{max}}{\beta^{DP}_{min}} \equiv 2. \quad (3.16)$$

When $GE \cdot e(nT) = L$ and $GR \cdot r(nT) = L$, β^L and β^{DP} become

$$\beta^L_{L,L} = \beta^{DP}_{L,L} = \frac{H \cdot GU}{2L}. \quad (3.17)$$

In addition to being symmetrical about the lines described in (3.3) and (3.4), β^L and β^{DP} are also symmetrical about the lines

$$GE \cdot e(nT) = 0 \quad (3.18)$$

$$GR \cdot r(nT) = 0. \quad (3.19)$$

It is concluded that starting from the minimum at $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$, β^L and β^{DP} strictly monotonically increase with increasing $GE \cdot |e(nT)|$ and $GR \cdot |r(nT)|$ in all directions. A three-dimensional plot of $\beta^L(\beta^{DP})$ for $\theta = 0$ is demonstrated in Fig. 6.

3.1.3. *Properties of* β^{BP}. For different θ, β^{BP} reaches different minimums. It either reaches

$$\beta^{BP}_{min} = \frac{(1+\theta)H \cdot GU}{2(1+3\theta)L} \quad (3.20)$$

when

$$GE \cdot e(nT) = GR \cdot r(nT) = 0 \quad (3.21)$$

or reaches

$$\beta^{BP}_{min} = \frac{\theta \cdot H \cdot GU}{(1+\theta)L} \quad (3.22)$$

when

$$GE \cdot e(NT) = L \quad \text{and} \quad GR \cdot r(nT) = -L. \quad (3.23)$$

Let the right side of equation (3.20) equal the right side of equation (3.22) and solve the new equation for θ. The solution is

$$\theta = \frac{\sqrt{5}}{5} \approx 0.4472 \quad (3.24)$$

which means when $0 \le \theta \le 0.4472$, β^{BP} reaches the minimum (3.22) and when $0.4472 \le \theta \le 0.5$, β^{BP} reaches the minimum (3.20).

It can be analytically proven that when

$$0 \le \theta \le 0.1827, \quad (3.25)$$

a variable β^{BP}_{max} takes place at

$$(GE \cdot e(nT),\ GR \cdot r(nT)) = \left(\frac{3 + 3\theta - \sqrt{-13\theta^2 + 6\theta + 3}}{2(1-\theta)}L,\ \frac{\sqrt{-13\theta^2 + 6\theta + 3} - 1 - \theta}{2(1-\theta)}L\right) \quad (3.26)$$

which strictly monotonically increases from $(0.6340L, 0.3660L)$ as $\theta = 0$ to $(L, 0.4472L)$ as $\theta = 0.1827$. When

$$0.1827 \le \theta \le 0.5, \quad (3.27)$$

another variable β^{BP}_{max} occurs at

$$(GE \cdot e(nT), GR \cdot r(nT)) = \left(L, \left(3 + \frac{\sqrt{6\theta^2 + 8\theta + 2} - 4}{1-\theta}\right)L\right) \quad (3.28)$$

which strictly monotonically increases from $(L, 0.4472L)$ as $\theta = 0.1827$ to $(L, 0.4772L)$ as $\theta = 0.5$. Because R_{BP} will be proven inappropriate for control purpose regardless of the expressions of β^{BP}_{max}, the expressions of β^{BP}_{max} are omitted and so is the range of ρ^{BP}.

It should be noted that when $GE \cdot e(nT) = L$ and $GR \cdot r(nT) = L$

$$\beta^{BP}_{L,L} = \frac{H \cdot GU}{2L}. \quad (3.29)$$

It is concluded that unlike the other βs, β^{BP}_{max} does not

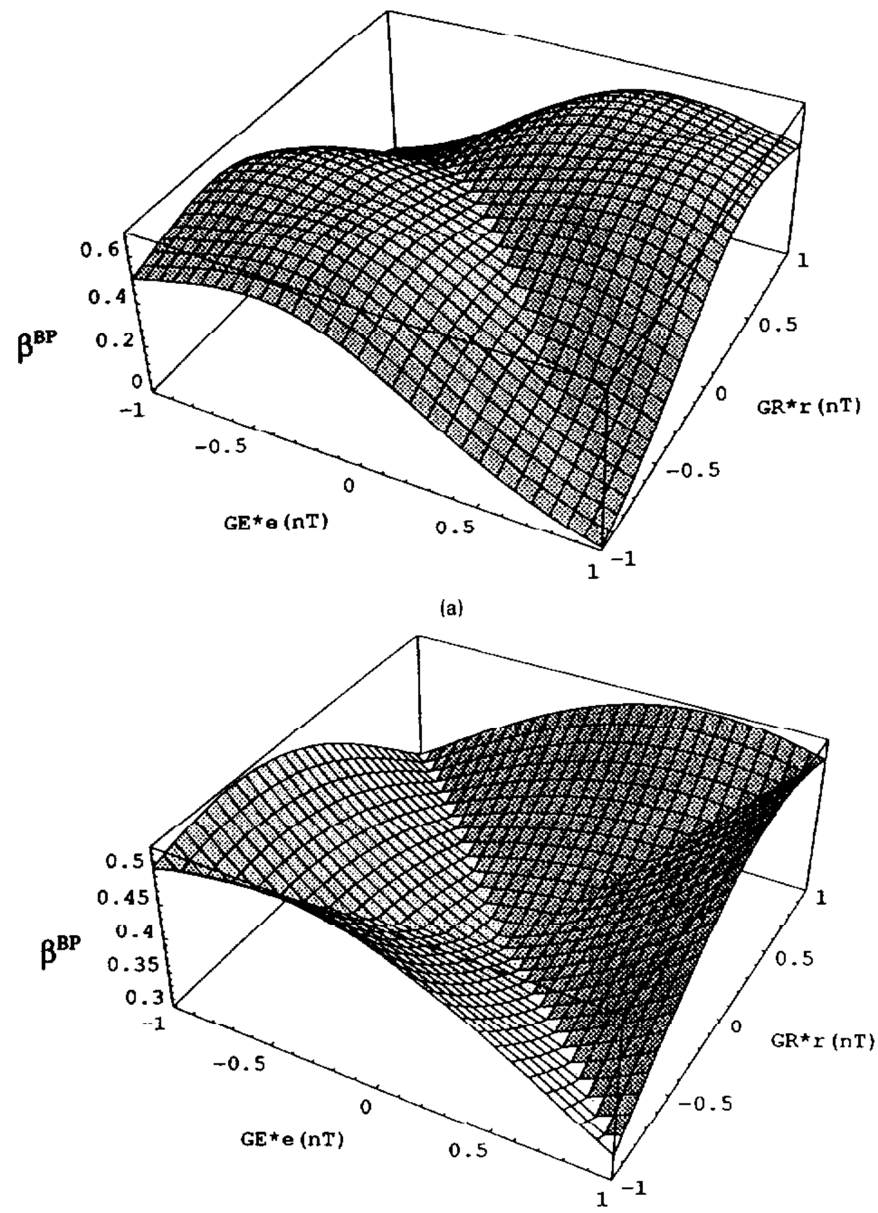


FIG. 7. Three-dimensional plots of β^{BP} for visualizing the properties of β^{BP} analyzed in the text. For the plots, $L = 1$, $GE = 1$, $GR = 1$, $H = 1$ and $GU = 1$. (a) $\theta = 0$. It can be seen that β^{BP}_{min} happens at $(L, -L)$ and $(-L, L)$, and β^{BP}_{max} takes place at $(0.634L, 0.366L)$, $(0.366L, 0.634L)$, $(-0.634L, -0.366L)$ and $(-0.366L, -0.634L)$. (b) $\theta = 0.5$. It can be seen that β^{BP}_{min} happens at $(0, 0)$, and β^{BP}_{max} takes place at $(L, 0.4772L)$, $(0.4772L, L)$, $(-L, -0.4772L)$ and $(-0.4772L, -L)$.

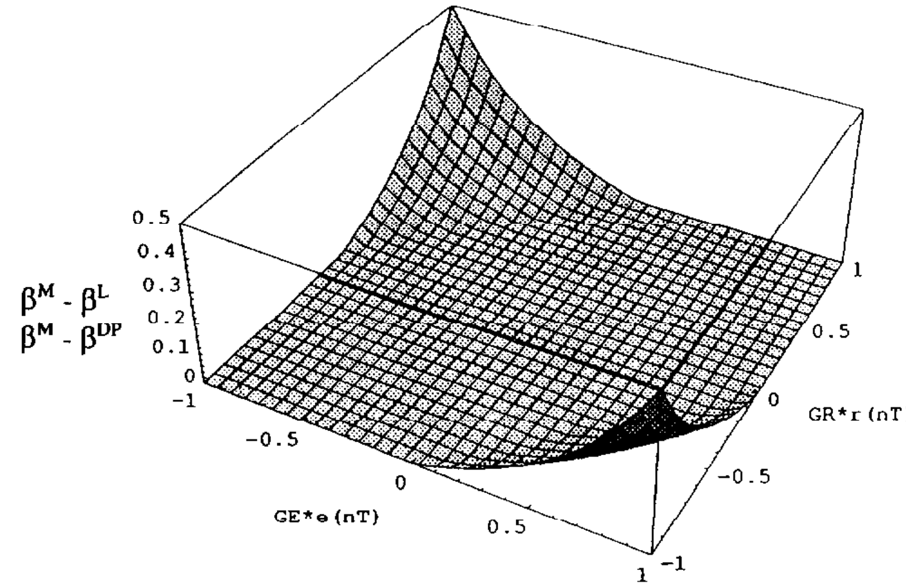
occur when $GE \cdot |e(nT)| = L$ and/or $GR \cdot |r(nT)| = L$. To visualize the theoretical discussion, two three-dimensional plots of β^{BP} for $\theta = 0$ and $\theta = 0.5$ are shown in Figs 7(a) and 7(b), respectively.

3.2. *Determination of the adequacy of the inference methods in the context of fuzzy control.*

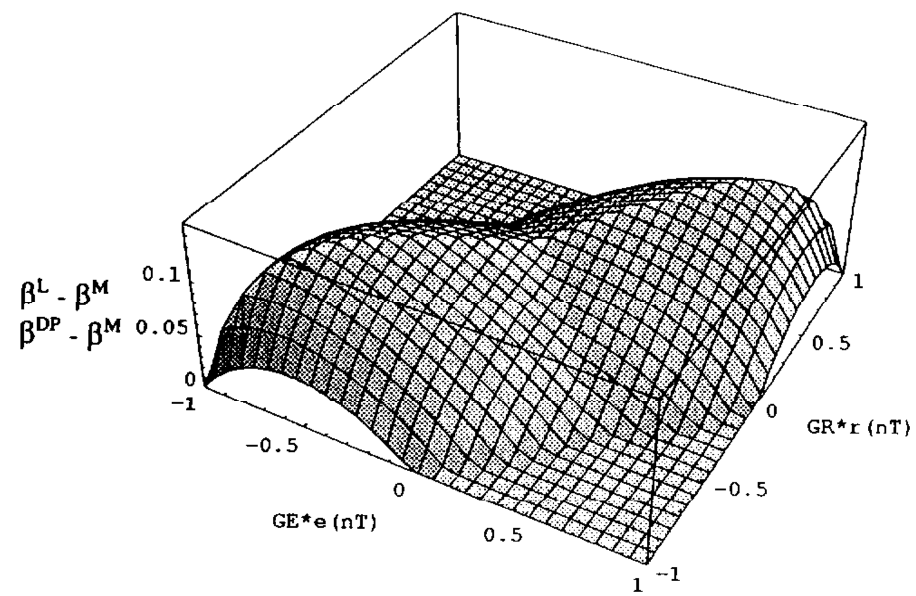
Theorem 2. For control purposes, R_M, R_L and R_{DP} work appropriately but R_{BP} does not.

Proof. When the absolute values of the scaled inputs are large, the fuzzy controllers using R_M, R_L and R_{DP} issue relatively strong control actions mainly due to the relatively large β^M, β^L and β^{DP} (the maximal control actions occur when $GE \cdot |e(nT)| = L$ and/or $GR \cdot |\gamma(nT)| = L$). The strong control actions quickly reduce large error of the process outputs with respect to the setpoint. On the other hand, when the absolute values of the scaled inputs are small, the fuzzy controllers issue relatively weak control actions because of relatively small β^M, β^L and β^{DP} (the minimal control actions take place when $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$). The weak control actions help gradually force the process outputs back to the setpoint in a manner that stabilizes the fuzzy control systems. From a control point of view, the behaviors of the fuzzy controllers are appropriate and desirable.

For the fuzzy controller using R_{BP}, the maximal control action does not happen at $GE \cdot |e(nT)| = L$ and/or $GR \cdot |r(nT)| = L$, which implies tha tthe fuzzy controller sometimes takes relatively strong action when the absolute



(a)

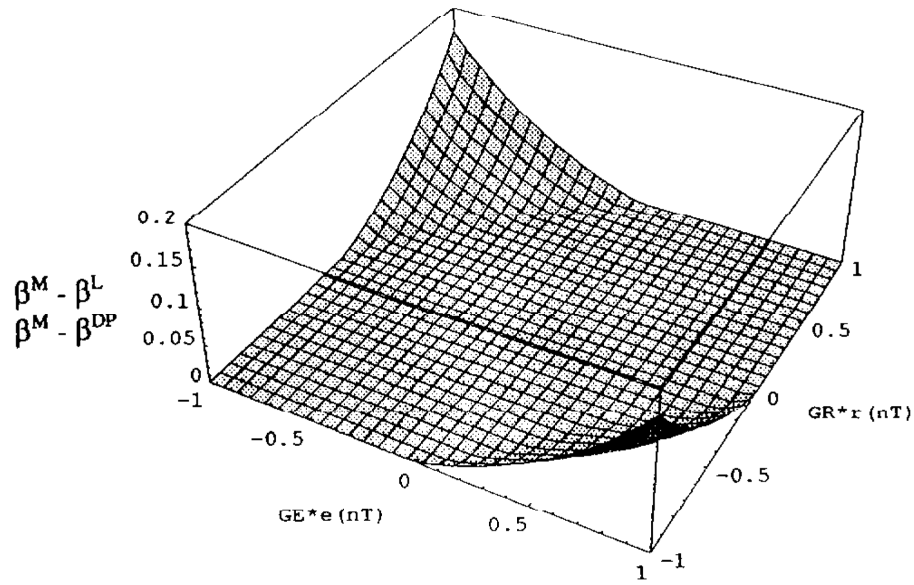


(b)

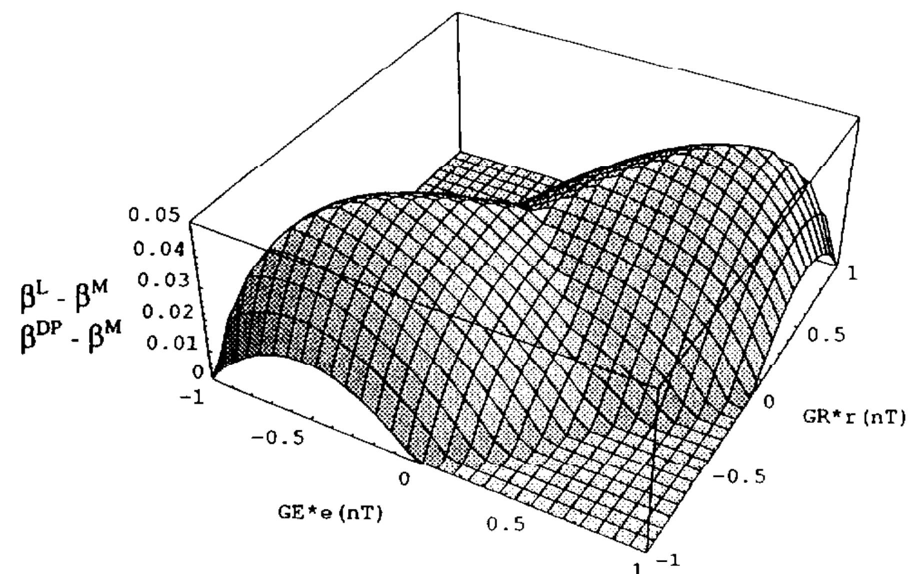
FIG. 8. Three-dimensional plots of $\beta^M - \beta^L(\beta^M - \beta^{DP})$ and $\beta^L - \beta^M(\beta^{DP} - \beta^M)$ for visualizing the differences between the control actions from the fuzzy controllers using R_M, R_L and R_{DP}. For the plots, $L = 1$, $GE = 1$, $GR = 1$, $H = 1$ and $GU = 1$. (a) Plot of $\beta^M - \beta^L(\beta^M - \beta^{DP})$ for $\theta = 0$. The surface means $\beta^M > \beta^L(\beta^{DP})$, which indicates that the control action from the fuzzy controller using R_M is stronger when the scaled inputs are in the areas near the coordinates $(L, -L)$ and $(-L, L)$. (b) Plot of $\beta^L - \beta^M(\beta^{DP} - \beta^M)$ for $\theta = 0$. The surface means $\beta^L > \beta^M(\beta^{DP} > \beta^M)$, which indicates that the control actions from the fuzzy controllers using R_L and R_{DP} are stronger when the scaled inputs are in, roughly speaking, either the first quadrant or the third quadrant of the $GE \cdot e(nT) - GR \cdot r(nT)$ plane. (c) Plot of $\beta^M - \beta^L(\beta^M - \beta^{DP})$ for $\theta = 0.5$. The surface means $\beta^M > \beta^L(\beta^{DP})$, which indicates that the control action from the fuzzy controller using R_M is stronger when the scaled inputs are in the areas near the coordinates $(L, -L)$ and $(-L, L)$. However, the differences between βs are significantly smaller than those when $\theta = 0$. (d) Plot of $\beta^L - \beta^M(\beta^{DP} - \beta^M)$ for $\theta = 0.5$. The surface means $\beta^L > \beta^M(\beta^{DP} > \beta^M)$, which indicates that the control actions from the fuzzy controllers using R_L and R_{DP} are stronger when the scaled inputs are in, roughly speaking, either the first quadrant or the third quadrant of the $GE \cdot e(nT) - GR \cdot r(nT)$ plane. However, the differences between βs are significantly smaller than those when $\theta = 0$.

values of the scaled inputs are small and takes relatively weak action when the absolute values of the scaled inputs are large. In other words, the magnitude of the control action is not always consistent with respect to the magnitude of the scaled inputs. The inconsistency is generally senseless and inappropriate from a control point of view. Additionally, the occurrence of β^{BP}_{min} at $(L, -L)$ and $(-L, L)$ is undesirable. Thus, R_{BP} should not be used for fuzzy control. ■

3.3. *Comparison of the control action of the fuzzy controller using R_M with those of the fuzzy controllers using*



(c)



(d)

FIG. 8. (*Continued*)

R_L and R_{DP}. Control action of the fuzzy controllers at sampling time nT is

$$GU \cdot \Delta u(nT) = -\beta(GE \cdot e(nT) + GR \cdot r(nT)), \quad (3.30)$$

which means the control action is calculated by both βs and the sum of the scaled inputs.

3.3.1. *When the fuzzy control systems are in transient state.* β^M_{max} takes place at the coordinates $(L, -L)$ and $(-L, L)$ while β^L_{max} and β^{DP}_{max} occur when $GE \cdot |e(nT)| = L$ or $GR \cdot |r(nT)| = L$. Since

$$\beta^M_{max} > \beta^L_{max} = \beta^{DP}_{max} \quad (3.31)$$

no matter what θ is, the control action of the fuzzy controller using R_M is always stronger than the other fuzzy controllers when the scaled inputs are in the area near the coordinates $(L, -L)$ and $(-L, L)$. On the other hand, because

$$\beta^M_{min} < \beta^L_{min} = \beta^{DP}_{min}, \quad (3.32)$$

there are some areas where the control actions of the fuzzy controllers using R_L and R_{DP} are stronger. Figures 8(a) and 8(b) illustrate graphically the difference between the control actions by showing the difference between βs when $\theta = 0$. Figure 8(a) is a surface of $\beta^M - \beta^L(\beta^M - \beta^{DP})$, cut at $\beta^M - \beta^L = 0$, to demonstrate when the control action of the fuzzy controller using R_M is stronger ($\beta^M > \beta^L$). Figure 8(b) is a surface of $\beta^L - \beta^M(\beta^{DP} - \beta^M)$, cut at $\beta^L - \beta^M =$, to show when the control actions of the fuzzy controllers using R_L and R_{DP} are stronger ($\beta^L > \beta^M$).

It can be seen from Fig. 8(a) that the control action of the fuzzy controller using R_M is larger when the process output of the fuzzy control system is far away from the setpoint but has a strong tendency to move back to the setpoint. Because $GE \cdot e(nT)$ and $GR \cdot r(nT)$ have opposite signs, the absolute value of the sum of $GE \cdot e(nT)$ and $GR \cdot r(nT)$ is small. Thus, the difference between the control actions of the fuzzy controllers is not significantly large, although β^M is quite a bit larger than β^L and β^{DP}.

In order to assess the impact of the control action on control performance, let us examine behavior of the fuzzy controllers when

$$\text{(a) } GE \cdot e(nT) \approx L \quad \text{and} \quad GR \cdot r(nT) \approx -L \tag{3.33}$$

$$\text{(b) } GE \cdot e(nT) \approx -L \quad \text{and} \quad GR \cdot r(nT) \approx L. \tag{3.34}$$

In these situations,

$$\beta^{M} \approx \beta^{M}_{max} \tag{3.35}$$

$$\beta^{L} \approx \beta^{L}_{max} \tag{3.36}$$

$$\beta^{DP} \approx \beta^{DP}_{max} \tag{3.37}$$

which maximizes the control actions of the controllers and consequently helps achieve rapid transient processes. It should be stressed that the control action of the fuzzy controller using R_M is stronger because of (3.31).

According to Fig. 8(b), the control actions of the fuzzy controllers using R_L and R_{DP} are stronger when the scaled inputs of the fuzzy control systems are primarily, roughly speaking, in the first or third quadrant of the $GE \cdot e(nT) - GR \cdot r(nT)$ plane. In these areas, the process outputs are (far) away from the setpoint and yet have a (strong) tendency to move further away from the setpoint. Because $GE \cdot e(nT)$ and $GR \cdot r(nT)$ have the same signs, the absolute value of the sum of $GE \cdot e(nT)$ and $GR \cdot r(nT)$ is large. Therefore, the difference between the control actions is more significant than the relatively small difference between β^M and $\beta^L(\beta^{DP})$ appears to suggest. The enlarged control actions quickly drive the process outputs back to the setpoint.

It should be mentioned that $\beta^M = \beta^L(\beta^M = \beta^{DP})$ at $(GE \cdot e(nT), \; GR \cdot r(nT)) = (L, L)$ and $(GE \cdot e(nT), GR \cdot r(nT)) = (-L, -L)$ because

$$\beta^{M}_{L,L} = \beta^{L}_{L,L} = \beta^{DP}_{L,L}. \tag{3.38}$$

For different θ, the surfaces of the differences between βs look similar but the scales are different. The smaller the θ, the larger the differences. Also, the areas on the $GE \cdot e(nT) - GR \cdot r(nT)$ plane corresponding to the surfaces of $\beta^M > \beta^L$ and $\beta^L > \beta^M$ are about the same for different θ. These facts can be verified by comparing Figs 8(a) and 8(b) with Fig. 8(c) and 8(d), respectively. These figures represent two extremes, in terms of θ, of the differences between βs.

Finally, it should be pointed out that ranges of K_p^M and K_i^M are always greater than those of $K_p^L(K_p^{DP})$ and $K_i^L(K_i^{DP})$ because

$$\rho^{M} > \rho^{L} = \rho^{DP}. \tag{3.39}$$

The difference can be as high as 300% when $\theta = 0$. Therefore, the triangular-shaped ($\theta = 0$) output fuzzy sets should be used for the fuzzy controller using R_M, if maximal dynamics and nonlinearity are needed.

3.3.2. *When the fuzzy control systems are in steady state.* When the fuzzy controllers are in steady state, $GE \cdot e(nT) \approx 0$ and $GR \cdot r(nT) \approx 0$. At that time

$$\beta^{M} \approx \beta^{M}_{min} \tag{3.40}$$

$$\beta^{L} \approx \beta^{L}_{min} \tag{3.41}$$

$$\beta^{DP} \approx \beta^{DP}_{min}. \tag{3.42}$$

Obviously, the control actions of the fuzzy controllers are minimized, which helps stabilize the control systems. Local stability of the fuzzy control systems at the equilibrium point (setpoint) can be actually determined by the following theorem.

Theorem 3 (Local Stability Theorem). The simplest fuzzy control systems using R_M, R_L and R_{DP} have the same local stability as the corresponding linear PI control system at the equilibrium point.

Proof. When $e(nT) = 0$ and $r(nT) = 0$, the fuzzy controllers represented by the nonlinear PI controllers with the variable gains become the linear PI controller with fixed gains. The gains are $GR \cdot \beta^M_{min}$ (or $GR \cdot \beta^L_{min}$ or $GR \cdot \beta^{DP}_{min}$) and $GE \cdot \beta^M_{min}$ (or $GE \cdot \beta^L_{min}$ or $GE \cdot \beta^{DP}_{min}$) for the fuzzy controller using R_M (or R_L or R_{DP}). According to Lyapunov's linearization method on stability [*e.g.* Slotine and Li (1991)], the nonlinear fuzzy control systems are asymptotically stable (or unstable) if the corresponding linear PI control system is stable (or unstable) at the equilibrium point. ■

The control action of the fuzzy controller using R_M is more conservative than those of the fuzzy controllers using R_L and R_{DP} because

$$\beta^{M}_{min} < \beta^{L}_{min} = \beta^{DP}_{min}. \tag{3.43}$$

Thus, the fuzzy controller using R_M is locally more stable than the fuzzy controllers using R_L and R_{DP} when the fuzzy controllers employ identical values of the parameters, such as θ, GE, GR and GU, *etc.*

On the basis of the transient state and steady state control action comparison analyses, the author concludes that drawing general conclusions on control performance comparison between the fuzzy controllers is difficult. Control performance is model-dependent and therefore careful case studies are necessary.

3.4. *Comparison of the control actions of the fuzzy controllers with that of the linear PI controller.* The linear PI controller described in (2.17) has constant proportional-gain and integral-gain, *i.e.* the gains do not change. The gains of the nonlinear PI controllers representing the fuzzy controllers always change with inputs. It is of interest to compare performances of the fuzzy controllers with that of the linear PI controller. It should be emphasized however that such a comparison should be made in a fair manner. To make a fair comparison, the most important thing to do is to make the fuzzy controllers and the linear PI controller comparable.

Let

$$K_p^{PI} = GR \cdot \beta_{min} \tag{3.44}$$

$$K_i^{PI} = GE \cdot \beta_{min} \tag{3.45}$$

where β_{min} represents β^M_{min}, β^L_{min} and β^{DP}_{min}. Then, the gains of the linear PI controller equal the gains of the nonlinear PI controllers when $GE \cdot e(nT) = 0$ and $GR \cdot r(nT) = 0$. The gains of the nonlinear PI controllers are always greater than those of the linear PI controller when $GE \cdot e(nT) \neq 0$ or $GR \cdot r(nT) \neq 0$. The author thinks that the equations (3.44) and (3.45) make the fuzzy controllers and the linear PI controller comparable and establish a fair ground for performance comparison between the two different types of controllers. This approach of performance comparison was adopted previously (Ying *et al.*, 1990). It was demonstrated that the fuzzy controller using R_{DP} performed significantly better than the linear PI controller, when a nonlinear process and a first-order process with a time delay were involved. When the linear processes were controlled, performances of the controllers were virtually identical. Analyses indicated that it was the variable gains that made the fuzzy controller perform superiorly.

On the basis of the properties of β^M and β^L and the control action comparisons between the fuzzy controllers in the above sections, the author concludes that the fuzzy controllers using R_M and R_L can also outperform the linear PI controller when nonlinear processes or processes with time-delay are involved. However, there seems to be little advantage to use the fuzzy controllers to control linear processes, no matter which inference methods are employed.

4. *The fuzzy controllers are adaptive controllers*

The resultant analytical structures of the fuzzy controllers are nonlinear adaptive PI controllers. Adaptation is achieved by automatically varying the proportional-gains and integral-gains of the nonlinear PI controllers to dynamically cope with different input states. In some sense, such kind of adaptation reflects adaptive behavior of human beings. The adaptation mechanism is *inherently* built in by the fuzzy control schemes, which is a natural consequence of emulating human control behavior.

There exist many adaptive controllers. To design an adaptive controller, an explicit mathematical model of a process to be controlled is usually required. In contrast, such a model is not necessary when a fuzzy controller is developed. It is well known that in practice, accurate process models, especially nonlinear models, are often hard to obtain. Therefore, when a process model is not available, as far as practicality is concerned (time, cost, implementation, *etc.*), model-free fuzzy controllers, many of which are adaptive controllers such as those in this paper (see also Ying, 1992b, 1993 for more cases) are more advantageous than model-dependent adaptive controllers. However, it is important to be aware that not every fuzzy controller is an adaptive controller. If one wants to find out whether a specific fuzzy controller is an adaptive controller, he/she will have to expose the analytical structure of the fuzzy controller in terms of adaptive control theory, which is a very difficult or even impossible task. Analytically modeling a fuzzy controller may require more effort than finding the reasonably accurate process model involved. On the other hand, when a process model is available, adaptive control techniques ought to be tried first, in this author's opinion, to take full advantage of relatively well-developed adaptive control theory which can provide comprehensive analysis and synthesis techniques, including stability analysis. Fuzzy controllers are nonfuzzy nonlinear controllers with possible inherent adaptation mechanisms. Fuzzy and nonfuzzy controllers cannot replace each other primarily because their design methods fundamentally differ. They are different controllers for different control problems.

5. *Conclusions*

It has been analytically proven that the simplest fuzzy controllers using R_M, R_L, R_{DP} and R_{BP} are different nonlinear PI controllers with variable proportional-gains and integral-gains. The simplest fuzzy controllers using R_L and R_{DP} have identical analytical structure. By analytically investigating the properties of βs, which represent the analytical structures of the fuzzy controllers, the author concludes that R_M, R_L and R_{DP} are the valid and appropriate inference methods for control purposes.

Rigorous mathematical analysis has proven that R_{BP} is invalid and inappropriate for fuzzy control.

Dynamic and static control behaviors of the fuzzy controllers have been analytically analyzed and compared with each other. It is determined that R_M and R_L (R_{DP}) have their own distinct advantages in terms of control actions. General conclusions on control performance comparison cannot be drawn.

Due to the properties of β^M and $\beta^L(\beta^{DP})$, the fuzzy controllers are expected to be able to outperform the linear PI controller when nonlinear processes or processes with time-delay are involved. The fuzzy controllers, however, are expected to perform similarly to the linear PI controller when linear processes are involved. It has been analytically proven that the fuzzy control systems have the same local stability at the equilibrium point as the corresponding linear PI control system.

All these theoretical results clearly demonstrate the importance and power of rigorous mathematical analysis in the development of solid fuzzy control theory. More importantly, the results prove that rigorous mathematical analysis can provide important insights where simulation studies fail.

Finally, it should be pointed out that structure of a fuzzy controller is determined by all of its components. More research needs to be conducted to expose analytical structures of fuzzy controllers with other configurations.

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