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Practical Design of Nonlinear Fuzzy Controllers with Stability Analysis for Regulating Processes with Unknown Mathematical Models*

HAO YING†

A practical design procedure with some guidelines for parameter tuning has been developed to systematically design nonlinear fuzzy controllers to control processes whose mathematical models are unknown, eliminating most of the trial-and-error efforts previously associated with construction of fuzzy control systems.

Key Words—Control system analysis; control system design; fuzzy control; nonlinear control systems; PID control; relay control; stability.

Abstract—A practical procedure has been developed to systematically design nonlinear fuzzy controllers to control processes whose mathematical models are unknown. The design procedure, which is developed based on the analytical and limiting structures of the nonlinear fuzzy controllers revealed previously, can easily be implemented in two steps. In the first step, designers use a linear proportional–integral (PI) controller to control the process to be controlled by the fuzzy controller. The proportional-gain and integral-gain of the PI controller are tuned to obtain an acceptable process output. In the second step, the values of all the functional parameters of the fuzzy controller are calculated based on the proportional-gain and integral-gain as well as on the process output of the tuned PI control system. If linear control rules are used, it is theoretically proven that such a nonlinear fuzzy control system has the same local stability as the PI control system. Guidelines for adequate fine adjustment of different components of the fuzzy controllers have been developed theoretically. The design procedure eliminates most of the tedious and time-consuming trial-and-error efforts required presently in constructing fuzzy controllers. A real-world example of designing a nonlinear fuzzy controller to regulate blood pressure in post-surgical patients is given to demonstrate the practicality and effectiveness of the design procedure.

1. INTRODUCTION

One of the fundamental problems in fuzzy control technology is that there lacks a design theory on fuzzy contollers. Many fuzzy controllers have been constructed, instead of systematically designed, case by case by using the trial-and-error method guided by designers' experiences on fuzzy control. The trial-and-error method is time-consuming and often fails, especially for complex systems such as biological systems. Most importantly, in some critical control systems like feedback control of blood pressure in post-surgical patients in the intensive care unit (Ying *et al.*, 1992), little trial-and-error effort could be tolerated. As fuzzy control technology plays an increasingly important role in industry, a practical design theory is urgently needed to reduce the cost and time required for developing products using fuzzy controllers.

Much efforts have been made to develop fuzzy controller design theory. The selection of different components was studied using phase plane analysis in conjunction with linguistic trajectory for a fuzzy controller employing fuzzy relation and composition (Braae and Rutherford, 1979a). In Braae and Rutherford (1979b), a fuzzy controller was linguistically modeled and its control rules were linguistically designed. Ray *et al.* (1984) studied L_2-stability and design of SISO system associated with fuzzy logic controller. Gottwald and Pedrycz (1985) investigated the elimination of a subset of inconsistent fuzzy control rules. Togai and Wang (1985) established a method to derive a fuzzy control strategy. Tang and Mulholland (1987) studied the relationship between a specific fuzzy controller and a linear PI controller when the quantization level of the input fuzzy sets was infinitely fine. Based on the crude relationship obtained, they constructed the fuzzy controller and compared its performance with that of the PI controller. Different aspects of constructing fuzzy controllers, such as completeness, consistency and interaction of control rules and robustness of

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† Department of Physiology and Biophysics, Biomedical Engineering Center, Office of Academic Computing, University of Texas Medical Branch, Galveston, TX 77555, U.S.A.

fuzzy controllers, were studied in a book by Pedrycz (1989). Kouatli and Jones (1990, 1991) used a manufacturing process as an example to show how to construct fuzzy controllers using fuzzimetric arcs as fuzzy sets. The choice of different fuzzy logic in the design of a fuzzy controller was addressed in Gupta and Qi (1991), which illustrated the effect of using different fuzzy logic on the performance of the fuzzy controllers. Recently, Abdelnour *et al.* (1992) used input and output mapping factors to design fuzzy controllers. Hwang and Lin (1992) developed a stability approach to fuzzy control design for nonlinear systems. Isaka and Sebald (1993) proposed an optimization approach for fuzzy controller design. Some fuzzy controllers were designed to solve real-world problems (e.g. Bare *et al.*, 1990; Liaw and Wang, 1992). Additionally, Tanaka and Sugeno (1992) studied the stability and design technique for a new type of fuzzy control system put forward by Takagi and Sugeno (1985). For a recent comprehensive review of fuzzy control technology, the reader is referred to the paper (Lee, 1990) and the references therein.

Summarily speaking, there does not exist yet a systematic design procedure following which a fuzzy controller with reasonably good performance can be quickly constructed with little trial-and-error effort. This is the case primarily because the reported research could not closely relate design of fuzzy controllers to their analytical structures which were unavailable.

The objective of this research is to develop a practical design procedure based on the analytical structure of fuzzy controllers, which allows designers to systematically construct, with little trial-and-error effort, an adequate fuzzy controller to satisfactorily control a process whose mathematical model is unknown. The paper is organized as follows: the previously revealed analytical structure of the fuzzy controllers is briefly described. A design procedure and some practical guidelines are then theoretically developed. A nonlinear fuzzy controller is illustratively designed to regulate blood pressure in postsurgical patients. Computer simulation is utilized to visualize the design process. Finally, the conclusions are presented.

2. ANALYTICAL ANALYSIS OF THE STRUCTURE OF FUZZY CONTROLLERS

2.1. *Components of the fuzzy controllers*

The fuzzy controllers to be designed in this study are the ones most widely used in practice, which consist of the following components [refer to Ying (1991, 1992a) for details].

(1) Two scaled inputs, e^* and r^*:

$$e^* = GE \cdot e(nT) = GE[SP - y(nT)] \quad (1)$$

$$r^* = GR \cdot r(nT) = GR[e(nT) - e(nT - T)], \quad (2)$$

where nT is the sampling time with T being the sampling period, $y(nT)$ is the process output, $e(nT)$ is the error of the process output and $r(nT)$ is the rate change of error of the process output (rate, for short). SP is the setpoint of the process output, and GE and GR are the input scalars.

(2) $N = 2J + 1$ identical equally spaced trapezoidal-shaped input fuzzy sets for both e^* and r^*. The definition of the membership functions is illustratively given in Fig. 1. E_i and R_j represent fuzzy sets for e^* and r^*, respectively, whose linguistic names (e.g. "large", "small") can be arbitrarily assigned. There are J fuzzy sets for positive $e^*(r^*)$, J fuzzy sets for negative $e^*(r^*)$, and one fuzzy set for near zero $e^*(r^*)$. The ratio $\theta = A/S$ ($2A$ and $2S$ are the lengths of upper-side and lower-side of the trapezoids respectively) is used to describe the shape of the trapezoids. The ratio is limited to $0 \leq \theta \leq 0.5$ to avoid overlap between two adjacent upper-sides. When $\theta = 0$, $A = 0$ and the trapezoid becomes a triangle. According to Fig. 1, the value of the membership functions is

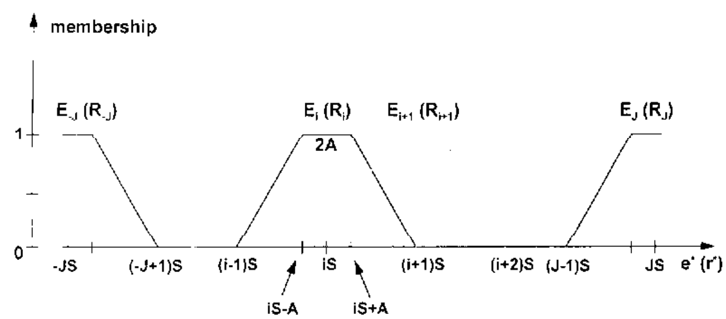

FIG. 1. Illustrative definition of the trapezoidal-shaped membership functions for fuzzifying e^* and r^*. The upper-side is $2A$ and the lower-side is $2S$. $L = J \cdot S$. When e^* and r^* are greater than L or less than $-L$, their corresponding memberships are always one.

between 0 to 1 in the interval $[-L, L]$, and one outside the interval.

There are $2N-1(4J+1)$ identical equally spaced output fuzzy sets for $\Delta u(nT)$, which denotes the incremental output of the fuzzy controllers (output, for short). U_k is employed to represent an output fuzzy set. There are $2J$ output fuzzy sets for positive $\Delta u(nT)$, $2J$ output fuzzy sets for negative $\Delta u(nT)$ and one output fuzzy set for near zero $\Delta u(nT)$. The output fuzzy sets are required to be normal and their membership functions are required to be symmetrical. The output fuzzy sets are defined over the interval $[-H, H]$. Consequently, the base of each output fuzzy set is $V = H/(2N-1)$.

(3) N^2 fuzzy control rules in the form

$$\text{IF } E^* \text{ is } E_i \text{ AND } r^* \text{ is } R_j \text{ THEN } \Delta u(nT) \text{ is } U_{f(i,j)}, \quad (3)$$

where $f(i, j)$ can be any function whose value at i and j is an integer, relating the indexes i and j of the input fuzzy sets to the index of the output fuzzy set. If f is a linear function, the fuzzy control rules generated by f are called linear fuzzy control rules (Buckley and Ying, 1989; Ying, 1993a).

(4) Larsen's product inference method and the drastic product inference method (Mizumoto, 1988; Ying, 1992b). These two inference methods use Zadeh fuzzy logic AND to calculate the membership of $U_{f(i,j)}$ from the memberships of E_i and R_j. All the output fuzzy sets generated by the control rules are directly used in defuzzification.

(5) The center of gravity defuzzification algorithm and an output scalar GU to scale $\Delta u(nT)$.

The above-defined components can be classified into two totally different types of parameters: the structural parameters and the functional parameters. The structural parameters, which determine the structure of the fuzzy controllers, are f, N, and θ. Generally speaking, it is difficult to theoretically calculate or determine exactly the structural parameters. They need to be determined empirically by designers case by case, mainly based on the type of process to be controlled, the knowledge of the process operators from whom the fuzzy control rules are obtained, and the experience of designers on fuzzy control technology. Some guidelines for determining N and θ will be provided later in this paper. It is important to point out that empirically selecting the structural parameters is a necessary step in fuzzy controller design, just like empirically selecting a specific structure of a controller (e.g. nonlinear controllers) is a necessary step in classical controller design. The functional parameters, which determine the system performance once the structural parameters are fixed, are GE, GR, GU, L and H.

Once the structural and functional parameters are determined, the remaining parameters, J, S, V and A, can be easily calculated.

2.2. *Analytical and limiting structures of the fuzzy controllers*

The analytical structure of the fuzzy controllers is the sum of a global f-dependent controller [denoted as $\Phi_G(i, j)$] and a local f-dependent PI-like controller [denoted as $\Phi_L(i, j)$]:

$$GU \cdot \Delta u(nT) = \Phi_G(i, j) + \Phi_L(i, j). \quad (4)$$

If $N \to \infty$, $\Phi_L(i, j) \to 0$ and $\Phi_G(i, j) \to \Phi_G^\infty$ which is an f-dependent nonlinear limiting controller (Ying, 1991, 1992a, 1993b).

When linear fuzzy control rules, represented by $f(i, j) = i + j$, are used, $\Phi_G(i, j)$ becomes a global two-dimensional multilevel relay while $\Phi_L(i, j)$ becomes a local nonlinear PI-like controller {or PI controller when $\theta = 0$ [Ying (1993a), also see Buckley and Ying (1989) on limiting structure of fuzzy controllers with linear control rules]}, as shown in Table 1. The input combinations (IC) appearing in the table are shown graphically in Fig. 2. Φ_G^∞ is a global linear PI controller in incremental form:

$$\Phi_G^\infty = \Delta u_{PI}(nT) = K_i \cdot e(nT) + K_p \cdot r(nT), \quad (5)$$

where the proportional-gain K_p and integral-gain K_i are

$$K_p = \frac{GR \cdot GU \cdot H}{2L}$$

and (6)

$$K_i = \frac{GE \cdot GU \cdot H}{2L}.$$

3. SYSTEMATIC AND PRACTICAL DESIGN OF THE FUZZY CONTROLLERS

3.1. *Design methodology*

Based on the above analytical and limiting structures of the fuzzy controllers, we propose to utilize the limiting controller, Φ_G^∞, as a means for the systematic and practial design of the fuzzy controllers. More specifically, the linear PI controller, which is Φ_G^∞ for the fuzzy controllers with the linear control rules, is utilized to develop a two-step procedure to design the fuzzy controllers with any type of fuzzy control rules. In the first step, designers use the linear PI controller to regulate the process to be controlled by the fuzzy controller. The proportional-gain and integral-gain of the PI

TABLE 1. THE SCALED INCREMENTAL OUTPUT OF THE FUZZY CONTROLLERS, $GU \cdot \Delta(nT)$, WHEN $f(i, j) = i + j$ IS USED TO GENERATE LINEAR CONTROL RULES AND WHEN BOTH e^* AND r^* ARE INSIDE THE INTERVAL $[-L, L]$. THE INPUT COMBINATIONS OF e^* AND r^* ARE SHOWN GRAPHICALLY IN FIG. 2.

Regions in which e^* and r^* lie	$\Phi_G(i, j)$	$\Phi_L(i, j)$
IC1, IC2, IC3, IC4, IC5, IC6, IC7 and IC8	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_1(\delta e^*, \delta r^*)(\delta e^* + \delta r^*)\frac{GU \cdot H}{2L}$
IC1′, IC4′, IC5′ and IC8′	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_2(\delta e^*, \delta r^*)(\delta e^* + \delta r^*)\frac{GU \cdot H}{2L}$
IC2′	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_2(\delta e^*, \delta r^*)[\delta e^* + (0.5S - A)]\frac{GU \cdot H}{2L}$
IC3′	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_2(\delta e^*, \delta r^*)[(0.5S - A) + \delta r^*]\frac{GU \cdot H}{2L}$
IC6′	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_2(\delta e^*, \delta r^*)[\delta e^* - (0.5S - A)]\frac{GU \cdot H}{2L}$
IC7′	$(i+j+1)\frac{GU \cdot H}{N-1}$	$\beta_2(\delta e^*, \delta r^*)[-(0.5S - A) + \delta r^*]\frac{GU \cdot H}{2L}$

Note:

$\delta e^* = e^* - (i + 0.5)S$

$\delta r^* = r^* - (j + 0.5)S$

$$\beta_1(\delta e^*, \delta r^*) = \begin{cases} \frac{S}{2S - 2|\delta e^*|}, & \text{for IC1, IC2, IC5 and IC6} \\ \frac{S}{2S - 2|\delta r^*|}, & \text{for IC3, IC4, IC7 and IC8} \end{cases}$$

$$\beta_2(\delta e^*, \delta r^*) = \begin{cases} \frac{S}{(2.5S - A) - 2|\delta e^*| - |\delta r^*|}, & \text{for IC1', IC2', IC5' and IC6'} \\ \frac{S}{(2.5S - A) - |\delta e^*| - 2|\delta r^*|}, & \text{for IC3' IC4', IC7' and IC8'} \end{cases}$$

controller are tuned to obtain an acceptable or better process output. In the second step, the values of the functional parameters, GE, GR, GU, L and H, are calculated based on the tuned proportional-gain and integral-gain as well as the process output of the tuned PI control system. The other parameters, J, S, V and A, are also calculated.

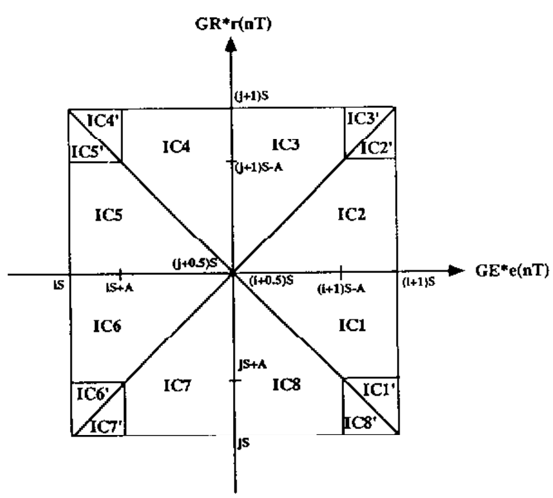

FIG. 2. Possible input combinations (IC) of e^* and r^* of process output when both e^* and r^* are within the interval $[-L, L]$. The explicit structure of the fuzzy controller in different IC is shown in Table 1.

Utilizing Φ_G^∞ corresponding to the linear control rules instead of nonlinear control rules is technically justified for developing the design procedure for the following reasons:

(1) Φ_G^∞ corresponding to nonlinear control rules is usually difficult or even impossible to obtain from given nonlinear control rules.

(2) The linear control rules represent a human operator's common-sense control strategy with respect to general processes. As an illustrative example, Table 2 shows 49 linear control rules. (See also Ying, 1993a).

TABLE 2. FORTY-NINE LINEAR CONTROL RULES GENERATED BY $f(i, j) = i + j$

	E_{-3}	E_{-2}	E_{-1}	E_0	E_1	E_2	E_3
R_{-3}	U_{-6}	U_{-5}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0
R_{-2}	U_{-5}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1
R_{-1}	U_{-4}	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2
R_0	U_{-3}	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3
R_1	U_{-2}	U_{-1}	U_0	U_1	U_2	U_3	U_4
R_2	U_{-1}	U_0	U_1	U_2	U_3	U_4	U_5
R_3	U_0	U_1	U_2	U_3	U_4	U_5	U_6

TABLE 3. FORTY-NINE NONLINEAR CONTROL RULES. THE CONTROL RULES THAT ARE DIFFERENT FROM THEIR LINEAR COUNTERPARTS SHOWN IN TABLE 2 ARE BOLD

	E_{-3}	E_{-2}	E_{-1}	E_0	E_1	E_2	E_3
R_{-3}	U_{-6}	$\mathbf{U}_{-6}$	U_{-4}	$\mathbf{U}_{-4}$	U_{-2}	U_{-1}	U_0
R_{-2}	U_{-5}	$\mathbf{U}_{-5}$	$\mathbf{U}_{-4}$	U_{-2}	$\mathbf{U}_{-2}$	U_0	U_1
R_{-1}	U_{-4}	$\mathbf{U}_{-4}$	$\mathbf{U}_{-3}$	$\mathbf{U}_{-2}$	U_0	$\mathbf{U}_2$	U_2
R_0	$\mathbf{U}_{-4}$	$\mathbf{U}_{-3}$	U_{-1}	U_0	U_1	$\mathbf{U}_3$	$\mathbf{U}_4$
R_1	U_{-2}	$\mathbf{U}_{-2}$	U_0	$\mathbf{U}_2$	$\mathbf{U}_3$	$\mathbf{U}_4$	U_4
R_2	U_{-1}	U_0	$\mathbf{U}_2$	U_2	$\mathbf{U}_4$	$\mathbf{U}_5$	U_5
R_3	U_0	U_1	U_2	$\mathbf{U}_4$	U_4	$\mathbf{U}_6$	U_6

(3) Although nonlinear control rules employed in practice are different from case to case, they usually have some common characteristics, as the 49 nonlinear control rules in Table 3 show. The nonlinear control rules are symmetric about the off-diagonal of the table, which is the case for the majority of fuzzy controllers reported in fuzzy control literature. Also, within a row, $\Delta u(nT)$ gradually increases from left to right, while within a column, $\Delta u(nT)$ gradually increases from top to bottom. Also note that $\Delta u(nT)$ corresponding to E_0 and R_0 is U_0 and $\Delta u(nT)$ corresponding to the central area of the table is small (e.g. U_{-1} and U_2). These characteristics of nonlinear control rules reflect both the general characteristics of a process and consistency of the operator's control action in general. Due to these common characteristics, a nonlinear f usually only mildly deviates from the linear f. Another factor preventing f from being severely nonlinear is that N is often quite small in practice.

(4) The linear PI controller can adequately be tuned with little trial-and-error effort guided by qualitative knowledge about the process and numerous theoretical and practical methods (e.g. Åström *et al.*, 1992). Usually, desirable control performance can be quickly achieved even if an explicit process model is unavailable. Considering the popularity and practicality of the PI controller, it is reasonable to use it in the design procedure even when Φ_G^∞ corresponding to nonlinear fuzzy control rules is available.

The design procedure has the following laudable properties.

3.2. *Properties of the design procedure*

Theorem 1.

$$|GU \cdot \Delta u(nT) - \Delta u_{PI}(nT)| \leq \frac{GU \cdot H}{2(N-1)}, \quad (7)$$

where $GU \cdot \Delta u(nT)$, described in Table 1, is the incremental output of the fuzzy controllers with the linear control rules and $\Delta u_{PI}(nT)$ is the incremental output of the linear PI controller in (5).

Proof. When e^* and r^* are in regions IC1 to IC8 (Fig. 2a), according to Table 1,

$$\begin{aligned} GU \cdot \Delta u(nT) &= \Phi_G(i, j) + \Phi_L(i, j) \\ &= (i + j + 1)\frac{GU \cdot H}{N-1} \\ &\quad + \beta_1(\delta e^*, \delta r^*)(\delta e^* + \delta r^*) \\ &\quad \times \frac{GU \cdot H}{2L}, \end{aligned} \quad (8)$$

where

$$\delta e^* = e^* - (i + 0.5)S, \quad \delta r^* = r^* - (j + 0.5)S, \quad -J \leq i, j \leq J - 1, \quad (9)$$

and

$$0.5 \leq \beta_1(\delta e^*, \delta r^*) \leq 1. \quad (10)$$

Therefore,

$$\begin{aligned} &|GU \cdot \Delta u(nT) - \Delta u_{PI}(nT)| \\ &= \frac{GU \cdot H}{N-1}\left|1 + \beta_1(\delta e^*, \delta r^*)\frac{\delta e^* + \delta r^*}{S} - \left(\frac{e^*}{S} - i\right) - \left(\frac{r^*}{S} - j\right)\right| \\ &= (1 - \beta_1(\delta e^*, \delta r^*))\frac{GU \cdot H}{N-1} \\ &\quad \times \left|1 - \left(\frac{e^*}{S} - i\right) - \left(\frac{r^*}{S} - j\right)\right|. \end{aligned} \quad (11)$$

Using minimum $\beta_1(\delta e^*, \delta r^*) = 0.5$, minimum $e^* = i \cdot S$ and minimum $r^* = j \cdot S + A$, the maximum of the equation (11) is achieved as

$$\frac{GU \cdot H(1 - \theta)}{2(N-1)}$$

which is less than

$$\frac{GU \cdot H}{2(N-1)}.$$

By the same token, the theorem can be proven true for the regions IC1′ to IC8′. ●

Theorem 1 provides the estimation for the difference between the incremental output of the fuzzy controllers with the linear control rules and that of the linear PI controller (Φ_G^∞). It discloses that if a fuzzy controller with (1) a very large N; and (2) GE, GR, GU, L and H such that K_p and K_i calculated in (6) are equal to the proportional-gain and integral-gain of the PI controller, then the fuzzy controller will perform like the PI controller. If N is relatively small, in a global sense, the fuzzy controller may act differently to a certain degree from what the PI controller does. However, in a region around the equilibrium point, *SP*, the two controllers should behave in a similar fashion according to the following theorem.

Theorem 2 (*local stability theorem*). For a given process, linear or nonlinear, the fuzzy control systems with the linear control rules have the same local stability (asymptotically stable or unstable) at the equilibrium point, *SP*, as the linear PI control system does. This also holds true for the fuzzy controllers with nonlinear control rules provided that f satisfies the following conditions:

$$f(0,0)=0,\quad f(0,-1)=-1,\quad f(-1,0)=-1$$

and

$$f(-1,-1)=-2. \tag{12}$$

Proof. To prove the theorem, we will show that the linearization of the nonlinear fuzzy controllers around the equilibrium point results in the linear PI controller.

When the process output $y(nT)$ is close to *SP*, $e(nT)\approx 0\ (e^*\approx 0)$ and $r(nT)\approx 0\ (r^*\approx 0)$. Therefore, e^* and r^* must be in the regions IC1 to IC8. There are four possible situations: (1) $e(nT)>0$ and $r(nT)>0$; (2) $e(nT)>0$ and $r(nT)<0$; (3) $e(nT)<0$ and $r(nT)>0$; and (4) $e(nT)<0$ and $r(nT)<0$. The corresponding i and j are: (1) $i=0$ and $j=0$; (2) $i=0$ and $j=-1$; (3) $i=-1$ and $j=0$; and (4) $i=-1$ and $j=-1$.

When $i=0$ and $j=0$, $\Phi_G(i,j)$ and $\Phi_L(i,j)$ in Table 1 become

$$\Phi_G(0,0)=\frac{GU\cdot H}{N-1} \tag{13}$$

and

$$\Phi_L(0,0)=\frac{[GE\cdot e(nT)-0.5S]+[GR\cdot r(nT)-0.5S]}{S}\times\frac{GU\cdot H}{N-1}. \tag{14}$$

Consequently, the output of the nonlinear fuzzy controllers becomes

$$GU\cdot\Delta u(nT)=\Phi_G(0,0)+\Phi_L(0,0)=\frac{GE\cdot GU\cdot H}{2L}e(nT)+\frac{GR\cdot GU\cdot H}{2L}r(nT) \tag{15}$$

which is the linear PI controller in (5). The same result can easily be obtained for the other three situations. Therefore, according to Lyapunov's linearization method on stability (e.g. Slotine and Li, 1991), the nonlinear fuzzy control systems are asymptotically stable (unstable) at the equilibrium point if and only if the linear PI control system is stable (unstable).

For the fuzzy controllers with nonlinear control rules, an f satisfying the conditions in (12) leads to the same stability result. ■

Stability is always the primary concern in design of any control system. It is advantageous to know precisely the size of the region around the equilibrium point ("basin") in which the nonlinear fuzzy control systems are locally stable. It is also desirable to determine the global stability of the fuzzy control systems. However, these tasks are difficult to accomplish because the fuzzy controllers are inherently nonlinear and the processes to be controlled are, practically speaking, nonlinear too. Global stability is often hard to determine theoretically and consequently computer simulation is usually necessary. Since the objective of this research is to produce a practical design procedure for the fuzzy controllers to control processes with unknown mathematical models, global stability analysis is unrealistic, and therefore is not pursued.

Based on these two theorems, one intuitively anticipates some similarity, in both the global sense and the local sense, between the process outputs of the nonlinear fuzzy control systems and that of the linear PI control system when N is not very large.

3.3. *Implementation of the design procedure*

3.3.1. *The first step.* Designers first appropriately choose the sampling period, T, according to the estimated characteristics of the process to be controlled. Designers then tune the linear PI controller to control the process whose mathematical model is unknown. The time response period of the system should be sufficiently long to cover both transient phase and a major part of steady state phase. Assume an acceptable or better process output is achieved. Here, the acceptable process output is required to avoid possible physical damage to the actual process. It is desirable, though not critical, to achieve as good a process output as possible. The better the process output of the tuned PI control system, the more appropriately the values of the functional parameters of the fuzzy controllers will be calculated, and hence less tuning efforts will be needed to adjust the calculated values of the functional parameters.

Let the proportional-gain and integral-gain be K_p^* and K_i^*, respectively, when the process output is acceptable or better. Also, assume the maximal absolute value of error and rate of the process output of the tuned linear PI control system are e_{max} and r_{max}, respectively.

3.3.2. *The second step.* Designers first calculate the values of the five functional parameters (GE, GR, GU, L and H) and the values of J, S, V and A. Designers then empirically select the structural parameters of the fuzzy controllers. The sampling period of the fuzzy control systems should be the same as that of the linear PI control system.

(1) Calculation of the values of the functional parameters, GE, GR and GU. The ratio of K_p and K_i in (6) is

$$\frac{GR}{GE}=\frac{K_p^*}{K_i^*}. \tag{16}$$

Obviously, if the value of GE is known, the value of GR can be calculated, or vice versa. The values of GE and GR are not critical but their ratio is. The value of $GE(GR)$ can be any number but for simplicity let $GE = 1$. The value of GR, therefore, can be calculated as

$$GR=\frac{K_p^*}{K_i^*}. \tag{17}$$

The value of GU can be calculated using (6) if the values of L and H are known. For simplicity, the value of H is assumed to equal to that of L. Such an assumption will be shown to be valid. When $L = H$, the value of GU can be calculated based on the equations in (6)

$$GU=\frac{2K_p^*}{GR}=2K_i^*. \tag{18}$$

(2) Calculation of the values of the functional parameters, L and H. It is desirable to choose a value of L such that most, if not all, of e^* and r^* will fall into the interval $[-L, L]$, which is achievable in our case if the value of L is calculated as

$$L=\mathrm{Max}\,\{GE\cdot e_{max}, GR\cdot r_{max}\}, \tag{19}$$

where $\mathrm{Max}\,\{\cdot\}$ is an operation of selecting the larger value among the two arguments. This is because the nonlinear fuzzy controller is expected to behave like the tuned linear PI controller in a global sense.

Next, the value of H needs to be calculated. Because GU and H always appear as a product in the analytical structure, as can be seen in Table 1, the value of H is not critical but the value of $GU\cdot H$ is. To simplify the above calculation of the value of GU, let

$$H=L. \tag{20}$$

(3) Calculation of the values of J, S, V and A. Obviously, $J=(N-1)/2$, $S=L/J$, $V=H/2J$, and $A=\theta\cdot S$.

(4) Empirical selection of the structural parameters, N, θ and f. Different values of N result in different nonlinearity. According to Theorem 1, the larger the value of N, the closer the nonlinear fuzzy controller is to a linear PI controller. Theoretically assessing the effect of different values of N on the performance of the fuzzy control systems is difficult and is also model-dependent. According to fuzzy control literature, the value of N is usually moderate, with a typical value ranging from three to 15.

Different values of θ also result in different nonlinearity. According to Table 1, different values of θ do not alter the structure of the global two-dimensional multilevel relay $\Phi_G(i, j)$, but do change the structure of the local PI-like controller $\Phi_L(i, j)$ [i.e. A and $\beta_2(\delta e^*, \delta r^*)$]. According to Fig. 2, for different values of θ, the size of the square configured by $[(i+0.5)S-A, (i+0.5)S+A]$ on the e^* axis and $[(j+0.5)S-A, (j+0.5)S+A]$ on the r^* axis is different and so is the size outside this square but inside the square configured by $[i\cdot S, (i+1)S]$ and $[j\cdot S, (j+1)S]$. As an extreme, $\Phi_L(i, j)$ changes from the local PI-like controller to the local PI controller if $\theta = 0$ (Ying, 1991, 1992a). Theoretically assessing the effect of different values of θ on the performance of the fuzzy control systems is difficult and is model-dependent. It appears reasonable to use $\theta = 0.25$ as a trial, which is the median value of the θ range, 0–0.5.

The fuzzy control rules (f), which reflect the knowledge and experience of the process operators, should obviously be determined by designers case by case.

3.3.3. *Practical guidelines for locally fine-tuning the functional parameters.* As with all control systems, especially nonlinear ones, the control performance of the fuzzy controllers with calculated functional parameters may not be the desired performance specified prior to the design, which may especially be true if the control performance of the tuned linear PI control system is poor. If this is the case, the calculated functional parameters of the fuzzy controllers need to be carefully fine-tuned to attain the desired performance.

It is critical to differentiate the amount of effort required to fine-tune the calculated functional parameters from the amount of effort required to blindly search the functional parameters using the trial-and-error method widely practised currently in fuzzy control technology. The former effort is much less than the latter, because the purpose of the fine-tuning is to only modify the calculated functional parameters locally in their neighborhood in the five dimensional parameter space while the purpose of the searching is to find globally the appropriate values of the functional parameters, without good initial values, in the entire parameter space.

The following practical guidelines, attained theoretically according to the structure of the fuzzy controllers, are helpful for the fine-tuning process.

(1) The value of L should not be modified first because (1) it is objectively calculated from e_{max} and r_{max}; and (2) increase of GE and GR is equivalent to decrease of L, and vice versa.
(2) The value of H needs not be modified, as adjustment of the value of GU is equivalent to adjustment of the value of H, as discussed previously.
(3) Theoretically assessing the role of θ is difficult while practical assessment requires a lot of trial-and-error efforts. Hence, the value of θ should be adjusted later, if necessary.
(4) Fuzzy control rules are given by human experts. Usually, they are reasonable and only minor modification is needed. It is wise to modify the control rules later because it is difficult to identify the individual control rules responsible for overall unsatisfactory control performance. More importantly, the effect of the control rules on the system performance is not only determined by the control rules themselves but also by the functional parameters, mainly GE, GR and GU. Without adequate values of these parameters, refinement of the control rules is troublesome.
(5) In conclusion, GE, GR and GU should be tuned first.

The role of GE, GR and GU can be theoretically and approximately estimated from analogy to the gains of the linear PI controller which is normally expressed as

$$\Delta u^*_{PI}(nT) = K_p\left[\frac{T}{T_i}e(nT) + r(nT)\right], \quad (21)$$

where T_i is the integral-time. Φ^∞_G in (5) can be rewritten as

$$\Phi^\infty_G = K_p\left[\frac{K_i}{K_p}e(nT) + r(nT)\right]. \quad (22)$$

Changing the value of GE only affects the integral control term [the $e(nT)$ term]. Increasing the value of GE causes the value of K_i in (22) to increase and is equivalent to decreasing the value of T_i in (21), which, according to the PID control theory, will result in a more oscillatory process output with less system stability. Decreasing the value of GE will produce the opposite effect to the process output. If the value of GE is too small, it takes a long time to eliminate steady-state error of the process output due to the too weak integral action.

Changing the value of GR only affects the proportional control term [the $r(nT)$ term]. Increasing the value of GR causes the value of K_p to increase, resulting in a more responsive process output with less rise-time but longer settling-time. A too large value of GR causes a too large value of K_p which destabilizes the system. Decreasing the value of GR causes the value of K_p to decrease which produces the opposite effects.

Changing the value of GU affects both the proportional control term and the integral control term. Changing the value of GU causes the values of K_p and K_i to change. Because the change of the value of K_p is cancelled out by the change of the value of K_i in the integral control term, changing the value of GU in (22) is equivalent to changing the value of K_p in (21). Therefore, increasing the value of GU produces a more responsive but less stable control system. Decreasing the value of GU produces the opposite result.

Changing the values of L and H is equivalent to changing the value of K_p in (21).

4. NUMERICAL EXAMPLE

Example. In this example, we will demonstrate the effectiveness and, most importantly, the practicality of the design procedure by designing a fuzzy mean arterial pressure (MAP) control system. The nonlinear control rules used are those shown in Table 3. Compared with the linear control rules in Table 2, there are 22 different control rules (doubly underlined) in Table 3. The difference is significant as it accounts for 45% of the total 49 control rules.

The control task is to lower high MAP (e.g. 140 mm Hg) in some open-heart surgery patients in cardiac surgical intensive care units quickly (within a few minutes) to a normal level (e.g. 80–100 mm Hg). A feedback control system is clinically desired to read MAP level every 10 s and compute infusion rate of a fast-acting vasodilator drug sodium nitroprusside (SNP) infused to the patients. The feedback control

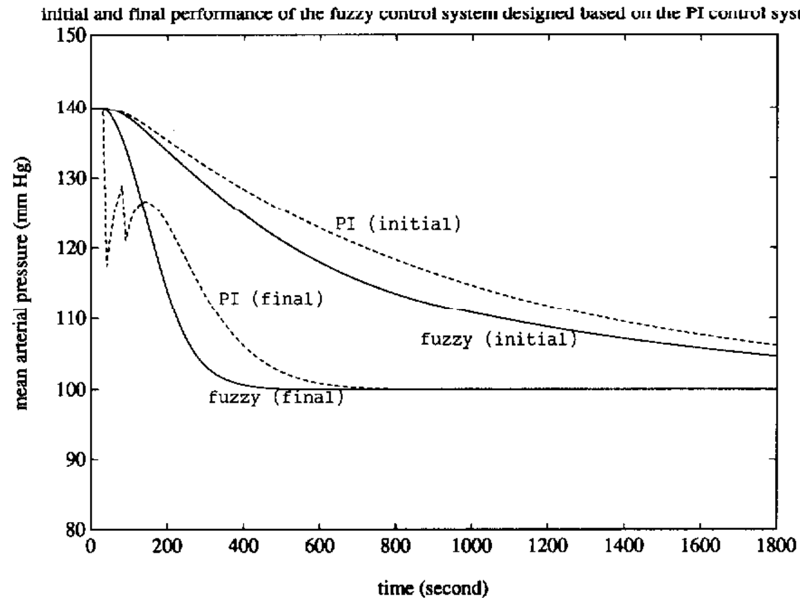

FIG. 3. The simulated performance of the tuned linear PI MAP control system and the initially designed nonlinear fuzzy MAP control system with 49 nonlinear control rules shown in Table 3. Also shown are (1) the final simulated performance of the fuzzy MAP control system after GU is fine-tuned; and (2) the simulated performance of the linear PI MAP control system using K_p and K_i calculated based on the tuned GU.

usually lasts for many hours until the patients' MAP can stay at the normal level without intervention of SNP.

The relationship between change of MAP, δMAP, and infusion rate of SNP is described by the following first-order transfer function (Slate and Sheppard, 1982):

$$\frac{\delta \text{MAP}(s)}{\text{SNP}(s)} = \frac{K \cdot e^{-30s}(1 + 0.4e^{-50s})}{1 + 40s}, \quad (23)$$

where K is a patient's sensitivity to SNP. For typical patients, $K = -0.72$. This model realistically represents patients and has therefore been widely used to design many nonfuzzy MAP control systems and a fuzzy MAP control system [see Ying *et al.* (1992) and the reference therein].

Supposedly we want to lower MAP from 140 mm Hg to 100 mm Hg for a given patient. Presumably, the poor performance of the tuned linear PI control system shown in Fig. 3 by the dotted line representing the initial PI control system performance is the best performance designers can achieve. It is found $K_p^* = -0.1$, $K_i^* = -0.01$, $e_{max} = 40$ and $r_{max} = 0.03725$.

Let $\theta = 0.25$ and $GE = 1$. The following parameters can be calculated. $GR = K_p^*/K_i^* = 10$ and $GU = 2K_i^* = -0.02$.

$$L = \text{Max}\{GE \cdot e_{max}, GR \cdot r_{max}\} = \text{Max}\{1 \times 40, 10 \times 0.03725\} = 40.$$

$H = L = 40$. $J = (N-1)/2 = (7-1)/2 = 3$ and $S = L/J = 13.333$. $V = H/2J = 6.667$ and $A = \theta \cdot S = 3.333$. The simulated performance of the fuzzy control system with these calculated parameter values is shown in Fig. 3 by the solid line representing the initial fuzzy control system performance. Obviously, the performance is unsatisfactory because MAP is reduced too slowly (the performance is clinically acceptable as far as patient's safety is concerned). Following the fine-tuning guidelines. The absolute value of GU is increased to improve system performance. A clinically very desirable performance is achieved, as shown in Fig. 3 by the solid line representing the final fuzzy control system performance, when $GU = -0.092$. It should be emphasized that such a good control performance is easily obtained by merely fine-tuning GU alone from the initial $GU = -0.02$ to the final $GU = -0.092$. This clearly demonstrates the practicality and effectiveness of the parameter calculation in the design procedure and how little effort is needed to locally fine-tuning the calculated parameters.

The performance of the linear PI control system whose gains are calculated based on the new $GU = -0.092$ is also plotted in Fig. 3. The new gains are: $K_p^* = GR \cdot GU/2 = -0.46$ and $K_i^* = GU/2 = -0.046$. Apparently, the performance is poor due to the oscillation in the transient phase which will likely cause too low MAP for patients who are very sensitive to SNP (i.e. $K = -2.88$), which could result in loss of patients' lives. In fact, it is a known fact that a linear PI controller cannot achieve clinically acceptable performance for all the patients.

5. CONCLUSIONS

Based on the theoretical analysis and the real-world example, we conclude that the design procedure and guidelines developed in this paper can easily and practically be used to systematically design nonlinear fuzzy controllers, with either linear or nonlinear control rules, without explicit mathematical models of the processes. The design procedure has greatly simplified the unmanageable task of blindly and globally searching the entire five (or more) dimensional parameter space to the much easier task of tuning a linear PI controller and calculating all the functional parameters based on the tuned PI control system, eliminating most of the trial-and-error efforts previously associated with construction of fuzzy control systems. The remaining little trial-and-error effort of locally fine-tuning some calculated parameters in their neighborhood parameter space is completely justified by the fact that the mathematical models of the processes are not required anywhere in the whole design process.

The design procedure can easily be automated. Optimal fine-tuning of the parameters under certain performance indexes can be achieved by optimization techniques. Another important benefit of using the design procedure is that the actual physical or biological processes have much less chance to be damaged since only little trial-and-error effort is needed.

The practical example demonstrates that an appropriately designed nonlinear fuzzy controller can significantly outperform the linear PI controller when a nonlinear or time-delay process is involved, which is consistent with our previous research results (Ying *et al.*, 1990).

The design procedure and guidelines are directly applicable to designing the fuzzy controllers using other types of input and output fuzzy sets, other types of fuzzy logic and other types of inference methods, because Φ_G^∞ is fundamentally determined by f alone (Ying, 1993b). Moreover, the principle of using the limiting controller as a means to develop design procedures should hold for much more general SISO and MIMO fuzzy controllers, because their analytical structure is still the sum of a global control-rules-dependent controller and a local controller (Ying, 1992b, 1993b). The limiting controller is always a control-rules-dependent controller. When linear control rules are used, the limiting controller is always a linear controller.

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