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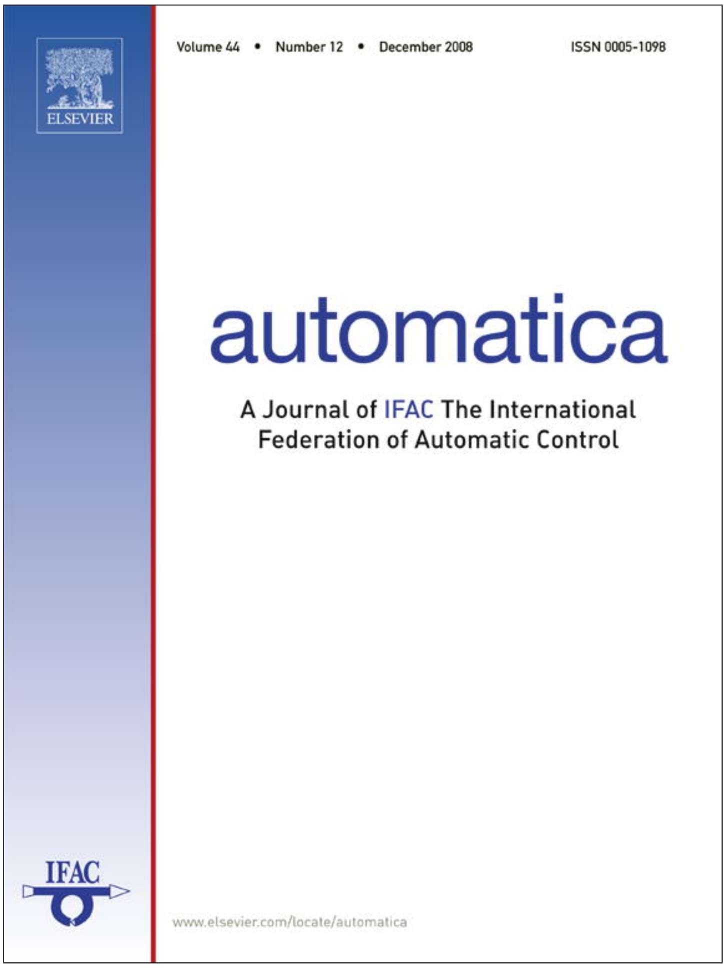

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Automatica 44 (2008) 3054–3060

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

State estimation and detectability of probabilistic discrete event systems☆

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ARTICLE INFO

Article history:
Received 22 January 2007
Received in revised form
27 March 2008
Accepted 19 May 2008
Available online 7 November 2008

Keywords:
Probabilistic discrete event systems
State estimation
Detectability
Observability

ABSTRACT

A probabilistic discrete event system (PDES) is a nondeterministic discrete event system where the probabilities of nondeterministic transitions are specified. State estimation problems of PDES are more difficult than those of non-probabilistic discrete event systems. In our previous papers, we investigated state estimation problems for non-probabilistic discrete event systems. We defined four types of detectabilities and derived necessary and sufficient conditions for checking these detectabilities. In this paper, we extend our study to state estimation problems for PDES by considering the probabilities. The first step in our approach is to convert a given PDES into a nondeterministic discrete event system and find sufficient conditions for checking probabilistic detectabilities. Next, to find necessary and sufficient conditions for checking probabilistic detectabilities, we investigate the "convergence" of event sequences in PDES. An event sequence is convergent if along this sequence, it is more and more certain that the system is in a particular state. We derive conditions for convergence and hence for detectabilities. We focus on systems with complete event observation and no state observation. For better presentation, the theoretical development is illustrated by a simplified example of nephritis diagnosis.

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1. Introduction

When applying discrete event system theory (Cao, Lin, & Lin, 1997; Cassandras & Lafortune, 1999; Cieslak, Desclaux, Fawaz, & Varaiya, 1988; Heymann & Lin, 1994; Kumar & Shayman, 1998; Lafortune & Lin, 1991; Li, Lin, & Lin, 1998; Lin & Wonham, 1988; Ozveren & Willsky, 1990; Ramadge, 1986; Ramadge & Wonham, 1987, 1989; Thistle, 1996; Wonham & Ramadge, 1988), we often need to know the state of a system. Clearly, if the initial state of a system is unknown, or if the system is nondeterministic, then we may not know the current state. In general, the state estimation is based on observations of events and state outputs. For example, by observing a copy machine, we can see a page being fed into the machine and its copy being made. But we may not be able to observe the movement of papers inside the copy machine. The events that can be seen are called observable events. Knowing what observable events have occurred can help us to determine the current state of the system. We may also observe some state outputs that can help us to obtain more information about the state of the system. For example, a signal warning that papers are jammed in the copy machine will tell us that some papers are stuck in the paper path, but it may not tell us how many papers are stuck or where they are stuck.

State estimation is always an important problem in system and control theory. In classical control theory, the state estimation problem is investigated in terms of observability, while in discrete event systems, the problem is investigated in terms of detectability. Although the state estimation of a discrete event system was investigated early in Ozveren and Willsky (1990) and Ramadge (1986), a full-scale study has not been done until our previous work (Shu, Lin, & Ying, 2006, 2007). We investigate the detectabilities of deterministic as well as nondeterministic discrete event systems in Shu et al. (2006, 2007). We say that a system is detectable if we can determine its state along some trajectories of the system. We say that a system is strongly detectable if we can determine its state along all trajectories of the system. We can check if a system is detectable or strongly detectable by constructing an observer, which summarizes the current estimate of the state of the system. In other words, by following the observer, we can say that after a sequence of observations, the system is in one among a set of possible states. If this set consists of only one state, then the state of the system is known exactly.

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Bart De Schutter under the direction of Editor Ian R. Petersen. This research is supported in part by NSF under grants INT-0213651 and ECS-0624828, and by NIH under grant 1 R21 EB001529-01A1.

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doi:10.1016/j.automatica.2008.05.025

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By constructing the observer, we can not only know the current estimate of the state, but also check detectability and strong detectability of the system.

Although we have investigated the detectabilities of deterministic as well as nondeterministic discrete event systems in Shu et al. (2006, 2007), there are some limitations of the previous work in dealing with some practical applications, especially in biomedical applications, where transition probabilities are important. Let us take nephritis for example. If we want to know if one person has nephritis or if the person has recovered from nephritis, the effective way is to check the urine sample. For a previously healthy person, if his/her urine sample is positive, then the person has developed nephritis recently. It is more difficult to check if a patient has recovered from nephritis. If his/her urine sample is positive, then the person still has nephritis. But if his/her urine sample is negative, it is possible that the patient has recovered from nephritis (we assume the probability is 0.7); or the patient may still have nephritis (we assume the probability is 0.3) because only repeated negative tests can confirm that the patient has recovered from nephritis. In order to model this formally, we define two discrete states:

Normal (q_1): the person is healthy

Failure (q_2): the person has nephritis

and two events:

$\tilde{\alpha}$: Urine test is negative

$\tilde{\beta}$: Urine test is positive.

Now, it is not difficult to derive the following probabilistic discrete event system (PDES) model (showed in Fig. 1) based on the reasons given above (a formal definition of PDES will be given shortly).

Since our goal is to determine whether a person is healthy or has nephritis; naturally, the problem can be translated into a state estimation problem of PDES.

Hence, in this paper, we consider the state estimation problem of probabilistic discrete event systems. We assume that a system is nondeterministic and assign probabilities to the nondeterministic transitions. The additional information on probabilities introduces more complexities as well as more opportunities to the state estimation problem. In particular, although we may never be 100% sure about the state of the system, we may be more and more certain about our estimation of the state as we observe more and more event occurrences. Therefore, a system may be undetectable in the non-probabilistic sense, but detectable in the probabilistic sense.

As one can expect, checking probabilistic detectabilities are much more complex than checking non-probabilistic detectabilities. Constructing an observer will not be sufficient for checking probabilistic detectabilities. We must study the asymptotical behavior of the system.

Markov chains are similar but not identical, to probabilistic discrete event systems. The state estimation problem has been investigated in Markov chain (Doucet & Andrieu, 1999; Doucet, Logothetis, & Krishnamurthy, 2000; Evans & Evans, 1997; Evans & Krishnamurthy, 1999, 2001; Ford & Moore, 1998; Habiballah, Ghosh-Roy, & Irving, 1998). However, the state estimation problems of Markov chains discussed in the literature are different from the estimation problem to be discussed in this paper, in at least the following two aspects: (1) While the observations in the estimation problems of Markov chains are some continuous variables and/or states, the observation in our estimation problem is the occurrence of events. Clearly, the observations in Markov chains are not event observations. In fact, the concept of events is not explicitly defined in Markov chains. Furthermore, observation noises are usually assumed in Markov chains but we do not have such an issue in our estimation problem. (2) States to be estimated are often not the states of the Markov chains but the state variables of some linear or nonlinear continuous systems built on the hidden Markov chains. To the best of our knowledge, the state estimation problem as defined in this paper has not been studied before.

We will first briefly review non-probabilistic discrete event systems and their detectabilities in Section 2, as our new results are built on the previous results in Shu et al. (2006, 2007). We will then define probabilistic discrete event systems and probabilistic detectabilities in Section 3. Again, two versions of detectabilities will be defined. The weak version requires the system be detectable for some trajectories of the system and the strong version requires the system be detectable for all trajectories of the system. To check detectability and strong detectability, in Section 4, we convert a probabilistic discrete event system into a non-probabilistic discrete event system, construct its observer, and investigate circular strings in the observer. For each circular string, we build its realizations to determine the system behavior along the string. By investigating the realizations, we can determine if the circular string converges or not. We can then determine the detectabilities of the system.

2. Non-probabilistic discrete event systems and detectabilities

We first briefly review the results on non-probabilistic detectability of discrete event systems studied in Shu et al. (2006, 2007). We use an automaton (also called state machine or generator) to model a discrete event system (Cassandras & Lafortune, 1999; Lin & Wonham, 1988; Ramadge & Wonham, 1987):

$$G = (Q, \Sigma, \delta),$$

where Q is the set of discrete states, Σ is the set of events, and δ is the transition function describing what event can occur at one state and the resulting new state. We assume that the system is nondeterministic, that is, the transition function is given by $\delta : Q \times \Sigma \rightarrow 2^Q$. Another equivalent way to define the transition function is to specify the set of all possible transitions: $\{(q, \sigma, q') : q' \in \delta(q, \sigma)\}$. With a slight abuse of notation, we will also use δ to denote the set of all possible transitions. We sometimes need to extend the definition of $\delta : Q \times \Sigma \rightarrow 2^Q$ to $\delta : 2^Q \times \Sigma \rightarrow 2^Q$: For a subset of states $x \subseteq Q$, the next set of possible states after σ is defined as $\delta(x, \sigma) = \cup_{q \in x} \delta(q, \sigma)$.

In this paper, we will assume that all events are observable and no states are observable. (In the terminologies of Shu et al. (2006, 2007), this means $\Sigma_o = \Sigma$ and $Y = \phi$.) We also assume that the system $G = (Q, \Sigma, \delta)$ is deadlock free, that is, for any state of the system, at least one event is defined at that state: $(\forall q \in Q)(\exists \sigma \in \Sigma)\delta(q, \sigma) \neq \phi$.

In Shu et al. (2006, 2007), the following state estimation problem is solved. Given a nondeterministic discrete event system, we do not know the initial state of $G = (Q, \Sigma, \delta)$. Under what conditions can we determine the current state of the system after a finite number of event observations?

Depending on whether we want to determine the current state for all trajectories or some trajectories and for all future times or some future times, we can define four types of detectabilities as shown in Table 1.

For example, strong detectability requires that we can determine the current state and the subsequent states of the system after a finite number of event observations for all trajectories of the system. This is the strongest version of detectability and only a few practical systems will be detectable in this strong sense. The procedure to check strong detectability can be summarized as follows.

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Table 1
Four types of detectabilities.

	For all trajectories	For some trajectories
For all future times	Strong detectability	(Weak) Detectability
For some future times	Strong periodic detectability	(Weak) Periodic detectability

In Step 1, we convert the nondeterministic automaton $G = (Q, \Sigma, \delta)$ into a deterministic automaton G_{obs} called observer using the standard method (Cao et al., 1997).

$$G_{obs} = (X, \Sigma, \xi, x_o) = Ac(2^Q, \Sigma, \xi, Q)$$

where $X \subseteq 2^Q$ is the state space,[1] $x_o = Q$ is the initial state, the transition function is given by $\xi(x, \sigma) = \{q \in Q : (\exists q' \in x)\ \delta(q', \sigma) = q\}$, and Ac denotes the accessible part. The automaton G_{obs} tells us which subset of states the system could be in after observing a sequence of events (that is, the state estimate).

In Step 2, we mark the states in G_{obs} that contain a singleton state of Q.

$$X_m = \{x \in X : |x| = 1\}$$

where $|x|$ denotes the number of elements in x.

In Step 3, we check if all loops in G_{obs} are entirely within X_m. If so, then the system can always reach X_m and stay within X_m after a finite number of observations (transitions). This is because of the assumption that the system is deadlock free. Therefore, the system is strongly detectable.

The procedures to check other detectabilities are similar and can be found in Shu et al. (2006, 2007).

The above defined detectabilities are non-probabilistic. They are independent of probabilities of event occurrences. Therefore, if a system is detectable, then we can determine the state of the system for sure after finite event observations. This version of detectabilities may be too restrictive because in many applications, especially in medical applications, we can never determine the state of the system for sure, but we can be more and more certain that the system is in a particular state. For such applications, we investigate probabilistic detectabilities. This requires the introduction of probabilistic discrete event systems. From now on, if it is clear from the context, we will call probabilistic detectabilities simply detectabilities.

3. Probabilistic discrete event systems and detectabilities

To generalize the model of non-probabilistic discrete event systems to that of probabilistic discrete event systems, we first reformulate non-probabilistic automata $G = (Q, \Sigma, \delta)$ and represent states, events, and their transitions by vectors and matrices: We enumerate the states of G as $k = 1, 2, \ldots n$, where $n = |Q|$ = the number of states in G. The current state of the system is represented by a vector

$$q = [p(q_k)]_{1\times n} = [\,p(q_1) \quad \ldots \quad p(q_n)\,],$$

where $p(q_k) \in \{0, 1\}$ indicates whether the system is currently in state k, that is, $\{q_k : p(q_k) = 1\}$ is the set of states the system may be in. Each event σ is represented by an event transition matrix

$$\sigma = [\sigma_{ij}]_{n\times n} = \begin{bmatrix} \sigma_{11} & \ldots & \sigma_{1n} \\ \ldots & & \\ \sigma_{n1} & \ldots & \sigma_{nn} \end{bmatrix}$$

where σ_{ij} equals 1 if σ is defined from state i to state j; and equals 0 otherwise. In this way, if the current state of the system is q and σ occurs in the system, then the next state will be $q\sigma$ in terms of matrix multiplication; that is, $\delta(q, \sigma) = q\sigma$. Note that, with a slight abuse of notation, we use q to denote both a state and the vector representing the state. Similarly, we use σ to denote both an event and the matrix representing the event transition.

A probabilistic automaton is denoted by

$$\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}),$$

where $\tilde{Q}$, $\tilde{\Sigma}$, and $\tilde{\delta}$ are extensions of Q, Σ, and δ respectively. They are defined as follows. $\tilde{q} \in \tilde{Q}$ is a probabilistic state vector (representing the estimation of the state)

$$\tilde{q} = [p(q_k)]_{1\times n} = [p(q_1) \quad \ldots \quad p(q_n)],$$

where $p(q_k) \in [0, 1]$ with $\sum_{k=1}^{n} p(q_k) = 1$ takes values between 0 and 1 and represents the probability for the system to be in state k. The equation $\sum_{k=1}^{n} p(q_k) = 1$ states that the sum of the probabilities in different states must be one. $\tilde{\sigma} \in \tilde{\Sigma}$ is a probabilistic event transition matrix

$$\tilde{\sigma} = [\tilde{\sigma}_{ij}]_{n\times n} = \begin{bmatrix} \tilde{\sigma}_{11} & \ldots & \tilde{\sigma}_{1n} \\ \ldots & & \\ \tilde{\sigma}_{n1} & \ldots & \tilde{\sigma}_{nn} \end{bmatrix}$$

where $\tilde{\sigma}_{ij} \in [0, 1]$ with $\sum_{j=1}^{n} \tilde{\sigma}_{ij} = 1$ or 0, for $1 \le i \le n$, represents the probability for the system to transit from state i to state j when event $\tilde{\sigma}$ occurs. Therefore, if the current state vector is $\tilde{q}$ and $\tilde{\sigma}$ occurs in the system, then the next state will be $\tilde{\delta}(\tilde{q}, \tilde{\sigma})$. Since some events may not be defined in some states, we allow $\sum_{j=1}^{n} \tilde{\sigma}_{ij} = 0$ for some i. Note that stochastic automaton was discussed in detail early in Paz (1971) and Rabin (1963) and our probabilistic discrete event system is a slight modification of stochastic automaton.

For a system modeled by a probabilistic automaton, our goal is to solve the following problem.

3.1. State estimation problem

Given a probabilistic discrete event system

$$\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}),$$

we do not know the initial state of $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$, but we can observe event occurrences. Under what conditions can we obtain a more and more accurate estimate of the current state of the system after more and more event observations?

By obtaining a more and more accurate estimate of the current state of the system, we mean the following: Consider an infinite sequence of events $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$, that is, $\tilde{s}$ is a string in an ω-language ($\tilde{s} \in \tilde{\Sigma}^{\omega}$) representing a possible trajectory of the system (Thistle & Wonham, 1994). Note that under the deadlock-free assumption, any (finite) sequence can be extended to an infinite sequence. Along the sequence $\tilde{s}$, the state estimation can be obtained as follows. Initially, we do not know which state the system is in. We assume that all states are equally likely initially; that is, $\tilde{q}_o = [1/n \quad \ldots \quad 1/n]$. After the occurrence and observation of $\tilde{\sigma}_1$, the likelihood of the system in each state is given by $\tilde{q}_o\tilde{\sigma}_1$. However, the sum of elements in $\tilde{q}_o\tilde{\sigma}_1$ may not be equal to 1, as the early state estimation may be wrong and hence $\tilde{\sigma}_1$ may not be defined at some state in the early estimation. To overcome this difficulty, we introduce a normalization operator $N(.)$ as

$$N(v) = N\left(\begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}\right) = \begin{cases} \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}/T & \text{if } T = v_1 + v_2 + \cdots + v_n \neq 0 \\ \text{undefined} & \text{otherwise.} \end{cases}$$

[1] We use the following standard notations for the states and the set of states in the observer: The set of states of the observer is denoted by capital letter X. Its element, that is, a state of the observer is denoted by small letter x. Note that since a state of the observer is a subset of states of the original automaton, x denotes the same object which can be interpreted as either a state of the observer or a subset of states of the original automaton.

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Then, the state estimation after $\tilde{\sigma}_1$ is $N(\tilde{q}_o\tilde{\sigma}_1)$. Note that the operator $N(.)$ has the following property[2]:

$N(N(\tilde{q}_o\tilde{\sigma}_1)\tilde{\sigma}_2) = N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2)$.

In other words, it is sufficient to apply $N(.)$ once. Using $N(.)$, we can estimate the state of the system after observing a sequence of events $\tilde{t} = \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k$ as

$\tilde{q} = N(\tilde{q}_o\tilde{t}) = N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k)$.

In other words, $\tilde{q} = \tilde{\delta}(\tilde{q}_o, \tilde{t}) = N(\tilde{q}_o\tilde{t})$. If the maximal element in $\tilde{q}$ approaches 1, then we are more and more certain that the system is in one particular state (corresponding to the maximum element). In this case, we say that $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ converges.

Formally, an infinite sequence $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ of $\tilde{G}$ converges if

$\max N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k) \to 1$, as $k \to \infty$.

With this definition of convergence, we can define strong (probabilistic) detectability as follows.

Definition 1 (*Strong Detectability*). A probabilistic discrete event system

$\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$

is strongly detectable if from the initial state $\tilde{q}_o = [1/n \quad \ldots \quad 1/n]$, all infinite sequences $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ of $\tilde{G}$ converge.

Strong detectability requires that all infinite sequences converge. If only some sequences converge, then we have (weak) detectability.

Definition 2 (*Weak Detectability*). A probabilistic discrete event system

$\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$

is weakly detectable (or simply detectable) if from the initial state $\tilde{q}_o = [1/n \quad \ldots \quad 1/n]$, there exists at least one infinite sequence $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ of $\tilde{G}$ that converges.

Before we derive conditions for checking detectability and strong detectability, let us first obtain some intuitions from the following example.

Example 1. Let us first consider the nephritis diagnosis system shown in Fig. 1. The system has two states and two events. The event transition matrices are

$$\tilde{\alpha} = \begin{bmatrix} 1 & 0 \\ 0.7 & 0.3 \end{bmatrix}, \qquad \tilde{\beta} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Assuming $\tilde{q}_o = [0.5 \quad 0.5]$, we can estimate the probabilistic state vector after observing various sequences of events as follows.

$N(\tilde{q}_o\tilde{\beta}) = [0 \quad 1]$

$N(\tilde{q}_o\tilde{\beta}\tilde{\beta}) = [0 \quad 1]$

$N(\tilde{q}_o\tilde{\beta}\tilde{\beta}\tilde{\beta}) = [0 \quad 1]$

$N(\tilde{q}_o\tilde{\alpha}) = [0.85 \quad 0.15]$

$N(\tilde{q}_o\tilde{\alpha}\tilde{\alpha}) = [0.955 \quad 0.045]$

$N(\tilde{q}_o\tilde{\alpha}\tilde{\alpha}\tilde{\alpha}) = [0.986 \quad 0.014]$

$N(\tilde{q}_o\tilde{\beta}\tilde{\alpha}) = [0.7 \quad 0.3]$

$N(\tilde{q}_o\tilde{\beta}\tilde{\alpha}\tilde{\beta}\tilde{\alpha}) = [0.7 \quad 0.3]$

$N(\tilde{q}_o\tilde{\beta}\tilde{\alpha}\tilde{\beta}\tilde{\alpha}\tilde{\beta}\tilde{\alpha}) = [0.7 \quad 0.3]$.

[2] Note that this property is different from that of idempotent, which means $N(N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2)) = N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2)$.

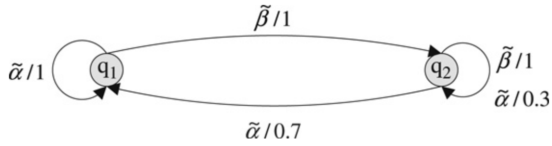

Fig. 1. The nephritis diagnosis system modeled by PDES.

Hence, we are certain that the system is in State 2 (having nephritis) after observations of event $\tilde{\beta}$ (positive urine test) and we are more and more certain that the system is in State 1 after more and more observations of event $\tilde{\alpha}$. On the other hand, if we keep observing string $\tilde{\beta}\tilde{\alpha}$, then we are not sure about the current state of the system. In other words, some infinite sequences converge but some others do not. Intuitively we know the system is detectable and not strongly detectable.

4. Checking detectabilities

To check detectabilities, we first convert the probabilistic automaton $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ into a non-probabilistic (and nondeterministic) automaton

$\text{convert}(\tilde{G}) = (Q, \Sigma, \delta)$

where $Q = \{q_1, q_2, \ldots, q_n\}$, $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_{|\tilde{\Sigma}|}\}$, and δ are defined as follows.

$(q_i, \sigma, q_j) \in \delta \Leftrightarrow \tilde{\sigma}_{ij} > 0$.

In other words, transition (q_i, σ, q_j) is defined in $\text{convert}(\tilde{G})$ if and only if the probability of event $\tilde{\sigma}$ occurring at state q_i and leading to state q_j is non-zero. δ can be extended to a mapping $\delta : 2^Q \times \Sigma \to 2^Q$ as described before. After converting $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$, we can take $G = \text{convert}(\tilde{G})$ and follow the steps outlined in Section 2 to obtain the observer automaton $G_{\text{obs}} = (X, \Sigma, \xi, x_o)$.

To see the connections between states and transitions of $\tilde{G}$ and G_{obs}, let us define, for a probabilistic state $\tilde{q} = [p(q_k)]_{1\times n} = [p(q_1) \quad \ldots \quad p(q_n)]$, its corresponding non-probabilistic state $\text{convert}(\tilde{q}) = \{q_i : p(q_i) \neq 0\}$. Hence, for $\tilde{q}_o = [1/n \quad \ldots \quad 1/n]$, $\text{convert}(\tilde{q}_o) = Q = x_o$.

Lemma 1. *Starting from the initial state $\tilde{q}_o = [1/n \quad \ldots \quad 1/n]$, if a sequence of events $\tilde{t} = \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k$ is observed, then the set of possible states the system may be in is given by*

$x = \xi(x_o, t) = \text{convert}(\tilde{\delta}(\tilde{q}_o, \tilde{t}))$,

where $t = \sigma_1\sigma_2 \ldots \sigma_k$ is the corresponding sequence in $\text{convert}(\tilde{G})$.

Proof. From our discussion in Section 3, the state of the system after observing a sequence of events $\tilde{t} = \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k$ in $\tilde{G}$ is given by

$N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k) = N(\tilde{q}_o\tilde{t}) = \tilde{\delta}(\tilde{q}_o, \tilde{t})$.

Therefore, we only need to prove $\xi(x_o, t) = \text{convert}(N(\tilde{q}_o\tilde{t}))$, which can be done by a simple induction on the length of $\tilde{t} = \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k$. □

Now we can present our first result.

Theorem 1. 1. *If $G = \text{convert}(\tilde{G})$ is non-probabilistically detectable, then $\tilde{G}$ is detectable.*
2. *If $G = \text{convert}(\tilde{G})$ is strongly non-probabilistically detectable, then $\tilde{G}$ is strongly detectable.*

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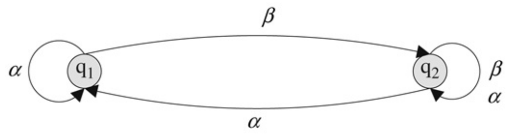

Fig. 2. The converted non-probabilistic and nondeterministic discrete event system from Fig. 1.

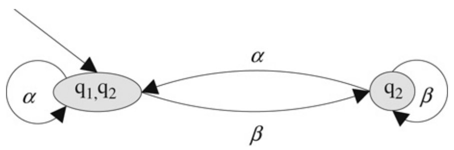

Fig. 3. The observer of the nondeterministic discrete event system in Fig. 2.

Proof. 1. If $G = \text{convert}(\tilde{G})$ is non-probabilistically detectable, then by the definition, the current state and the subsequent states of the system can be determined after finite number of event observations for some trajectories of the system. In other words, there exists an infinite string $\sigma_1\sigma_2 \ldots \sigma_k \ldots$ such that for sufficiently large k, $|\xi(x_o, \sigma_1\sigma_2 \cdots \sigma_k)| = 1$. By Lemma 1,

$$\begin{aligned}\xi(x_o, \sigma_1\sigma_2 \cdots \sigma_k) &= \text{convert}(\tilde{\delta}(\tilde{q}_o, \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k)) \\ &= \text{convert}(N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k)).\end{aligned}$$

Therefore, $|\xi(x_o, \sigma_1\sigma_2 \cdots \sigma_k)| = 1$ implies that there is only one element in $N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k)$ that is non-zero and it must be 1. Hence $\max N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k) = 1$ and $\tilde{G}$ is detectable

2. Similar proof but for all trajectories of the system. □

To check if $G = \text{convert}(\tilde{G})$ is non-probabilistically detectable or strongly non-probabilistically detectable, the results of Shu et al. (2006) can be used.

Example 2. For the probabilistic automaton $\tilde{G}$ shown in Fig. 1, the converted nondeterministic automaton $G = \text{convert}(\tilde{G})$ is given in Fig. 2.

Using the procedure of Section 2, we can compute the observer automaton G_{obs}, which is shown in Fig. 3.

From G_{obs}, we can see that $G = \text{convert}(\tilde{G})$ is (weak) non-probabilistically detectable but not strongly non-probabilistically detectable. By Theorem 1, $\tilde{G}$ is (weak) detectable. We do not know if it is strongly detectable because the conditions in Theorem 1 are only sufficient but not necessary. This can be seen as follows: From Fig. 3, we know that the event sequence $\alpha\alpha\alpha\cdots$ does not lead to a singleton state. However, the corresponding probabilistic event sequence $\tilde{\alpha}\tilde{\alpha}\tilde{\alpha}\cdots$ is convergent.

To obtain necessary and sufficient conditions for checking detectabilities, we need to consider infinite sequences generated by G_{obs}. These strings must form circles in G_{obs}. Let us define circular strings in G_{obs} that start and end at the same state $x \in X$ as

$$L_c(G_{\text{obs}}, x) = \{v \in \Sigma^* : \xi(x, v) = x\}.$$

The set of all circular strings in G_{obs} is given by

$$L_c(G_{\text{obs}}) = \cup_{x\in X} L_c(G_{\text{obs}}, x).$$

We denote a circular string from $x \in X$, $v = \sigma_1\sigma_2 \ldots \sigma_j \in L_c(G_{\text{obs}}, x)$ as a pair (x, v), its realization is a sequence

$$R(y_0, v) = (y_0 \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} y_2 \rightarrow \cdots y_{j-1} \xrightarrow{\sigma_j} y_j).$$

where $y_0 \subseteq x \subseteq Q$, $y_1 = \delta(y_0, \sigma_1)$, $y_2 = \delta(y_1, \sigma_2), \ldots, y_j = \delta(y_{j-1}, \sigma_j)$ such that $y_j \subseteq y_0$. A realization has the following property: if the system $G = \text{convert}(\tilde{G})$ starts within states y_0, then after the realization, the system remains within states y_0. A realization is nonblocking if for all $i = 0, 1, \ldots, j-1$ and for all $q \in y_i$, $\delta(q, \sigma_{i+1}) \neq \phi$.

If we start from $y_0 = x$, then by the definition of G_{obs}, $\xi(x, v) = x$. Therefore,

$$R(x, v) = (x \xrightarrow{\sigma_1} x_1 \xrightarrow{\sigma_2} x_2 \rightarrow \cdots x_{j-1} \xrightarrow{\sigma_j} x_j).$$

is a realization with $x_j \subseteq x$. However, $R(x, v)$ may block. Also, it may not be minimal. A realization $R(y, v)$ is a minimal realization if there exists no other realization $R(y', v)$such that $y' \subset y$. To find realizations and minimal realizations, we construct a transition graph

$$TG(x, v) = (x, \text{Arc}(v, x))$$

for $v \in L_c(G_{\text{obs}}, x)$ as follows. The nodes of $TG(x, v)$ are x. For $q_1, q_2 \in x$, the arc $(q_1, q_2) \in \text{Arc}(v, x)$ is defined if and only if $q_2 \in \delta(q_1, v)$. In other words, $(q_1, q_2) \in \text{Arc}(v, x)$ means that starting from state $q_1 \in x$, it is possible for the system to end at state $q_2 \in x$. If the system starts at a subset of states $y \subseteq x$, then the set of possible states after v is

$$\text{Post}(y) = \{q_2 \in x : (\exists q_1 \in y)(q_1, q_2) \in \text{Arc}(v, x)\}.$$

From these definitions, we can check a realization and nonblocking realization as follows.

1. $R(y_0, v) = (y_0 \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j)$ is a realization if and only if $\text{Post}(y_0) \subseteq y_0$.
2. $R(y_0, v) = (y_0 \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j)$ is a nonblocking realization if and only if it is a realization and for all $i = 0, 1, \ldots, j-1$ and for all $q \in y_i$, $\delta(q, \sigma_{i+1}) \neq \phi$.

Checking minimal realization is more involved and needs the concept of strongly connected subgraphs. A subgraph with states $y \subseteq x$ is strongly connected if and only if there exists a direct path from any state to any other state in y. For a subgraph with only one node, it is strongly connected if and only if the node has a self-loop. Strongly connected subgraphs are related to minimal realizations as follows.

Lemma 2. *$R(y, v)$ is a minimal realization if and only if $\text{Post}(y) \subseteq y$ and the subgraph of y is strongly connected.*

Proof. If y has only one state, then the result is obvious. If y has more than one states, then we can partition states or remove states to prove the result as follows.

If the subgraph of y is not strongly connected, then we can partition it into two subgraphs y_1 and y_2 and there is no direct path from states in y_1 to states in y_2. We can remove states in y_2 and the remaining subgraph still satisfies $\text{Post}(y_1) \subseteq y_1$. Therefore, $R(y_1, v)$ is a realization and $R(y, v)$ is not a minimal realization.

On the other hand, if the subgraph of y is strongly connected, then removing any subset y' from y will result in $\text{Post}(y \setminus y') \subseteq (y \setminus y')$ being no longer true. □

Given a pair (x, v) with $v = \sigma_1\sigma_2 \ldots \sigma_j \in L_c(G_{\text{obs}}, x)$, we say that it converges if it has a unique minimal and nonblocking realization

$$R(y, v) = (y \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j)$$

such that

$$|y| = |y_1| = \cdots = |y_j| = 1.$$

$R(y, v)$ is called the convergent realization of (x, v) and denoted by $R_c(y, v)$. Therefore, a pair (x, v) converges if and only if it has the (unique) convergent realization $R_c(y, v)$.

Given a circular string $v = \sigma_1\sigma_2 \ldots \sigma_j \in L_c(G_{\text{obs}})$, we say that $v = \sigma_1\sigma_2 \ldots \sigma_j$ converges if there exists a convergent pair (x, v); and $v = \sigma_1\sigma_2 \ldots \sigma_j$ strongly converges if all the pairs (x, v) are convergent.

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Theorem 2. *A PDES $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is detectable if and only if the corresponding $G = \text{convert}(\tilde{G})$ and G_{obs} have at least one convergent circular string $v \in L_c(G_{obs})$.*

Proof. (IF) Assume that $G = \text{convert}(\tilde{G})$ and G_{obs} has a convergent circular string $v = \sigma_1\sigma_2 \ldots \sigma_j \in L_c(G_{obs}, x)$ whose convergent realization is

$$R(y, v) = (y \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j).$$

By the definition, $R(y, v)$ is the unique minimal and nonblocking realization of $v \in L_c(G_{obs}, x)$ and $|y| = |y_1| = \cdots = |y_j| = 1$.

Let $u \in \Sigma^*$ be a string such that $\xi(x_o, u) = x$ (u exists because x is accessible). By Lemma 1, we have $x = \text{convert}(\tilde{\delta}(\tilde{q}_o, \tilde{u}))$. Let $\tilde{q} = \tilde{\delta}(\tilde{q}_o, \tilde{u})$ and $\tilde{v} = \tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_j$; that is, $\tilde{q}$ and $\tilde{v}$ are the state set and the string in $\tilde{G}$ corresponding to x and v in G. We prove that when $\tilde{v}$ repeats infinitely, the infinite sequences $\tilde{s} = \tilde{u}\tilde{v}\tilde{v} \ldots \tilde{v} \ldots$ of $\tilde{G}$ converge. Since $|y| = 1$, let us write $y = \{q_i\}$, that is, y is the ith state. Because $R(y, v)$ is a nonblocking realization, $p_i(N(\tilde{q}\tilde{v}^m)) \geq p_i(\tilde{q})$ and $\lim_{m\to\infty} p_i(N(\tilde{q}\tilde{v}^m)) \geq p(q_i)$. Because $y \subseteq x$ and there are no other minimal and nonblocking realizations, for all other states q_j, $\{q_j\} \neq y$, $\lim_{m\to\infty} p_j(N(\tilde{q}\tilde{v}^m)) = 0$. Hence,

$$\lim_{m\to\infty} \frac{p_i(\tilde{q}\tilde{v}^m)}{\sum_{k=1}^{n} p_k(\tilde{q}\tilde{v}^m)} = \lim_{m\to\infty} \frac{p_i(\tilde{q}\tilde{v}^m)}{p_i(\tilde{q}\tilde{v}^m)} = 1.$$

In other words,

$$\max N(\tilde{q}_o\tilde{u}\tilde{v}^m) \to 1, \quad \text{as } m \to \infty.$$

So $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is detectable. This proves the "IF" part.

(ONLY IF). Assume that $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is detectable. Then there exists an infinite sequence $\tilde{s}$ of $\tilde{G}$ that converges. Since the corresponding s is an infinite string in $G = \text{convert}(\tilde{G})$ and G_{obs}, it must visit some state x of G_{obs} infinitely often.

For a state $q \in x$, let $p^m(q)$ be the probability of $\tilde{G}$ being at q when $\tilde{s}$ visits x for the mth time. Since $\tilde{s}$ converges, there must exist a unique state $q \in x$ such that $p^m(q) \to 1$ as $m \to \infty$. Let m be sufficiently large and ε be sufficiently small and $p^{m+1}(q) \geq p^m(q) \geq 1 - \varepsilon$. Let $v = \sigma_1\sigma_2 \ldots \sigma_j$ be the substring of s between the mth and $(m+1)$th visit of x by G_{obs}. Clearly $v \in L_c(G_{obs}, x)$ is a circular string. We want to show that v converges and $R(\{q\}, v) = (\{q\} \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j)$ is the convergent realization with $y_j = \{q\}$.

To do so, let us first show that $R(\{q\}, v)$ is a realization; that is, $y_j \subseteq \{q\}$ as follows. If $y_j \nsubseteq \{q\}$, then when the system starts at $\{q\}$ and after the occurrence of $v = \sigma_1\sigma_2 \ldots \sigma_j$, it may move outside of $\{q\}$, hence the probability of returning to $\{q\}$ after v is less, contradicting the assumption $p^{m+1}(q) \geq p^m(q) \geq 1 - \varepsilon$. $R(\{q\}, v)$ is minimal because $\{q\}$ consists of only one state. Also, $|y_1| = \cdots = |y_j| = 1$ must be true because $\tilde{s}$ converges. Also $R(\{q\}, v)$ is nonblocking because any y_i is a singleton state. Furthermore, $R(\{q\}, v)$ is the unique realization that is minimal and nonblocking, because otherwise, $q \in x$ is not the unique state such that $p^m(q) \to 1$ as $m \to \infty$. All these facts together show that $v \in L_c(G_{obs}, x)$ is a convergent circular string. This proves the "ONLY IF" part. □

The above theorem gives a necessary and sufficient condition for checking (weak) detectability. Now let us present the following theorem for a necessary and sufficient condition for checking strong detectability.

Theorem 3. *A PDES $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is strongly detectable if and only if all circular strings $v \in L_c(G_{obs})$ of the corresponding $G = \text{convert}(\tilde{G})$ and G_{obs} strongly converge.*

Proof. (IF) Assume that all circular strings $v \in L_c(G_{obs})$ of the corresponding $G = \text{convert}(\tilde{G})$ and G_{obs} strongly converge. Let us prove that $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is strongly detectable by contradiction. Suppose that $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ is not strongly detectable. Then there exists an infinite sequence $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ of $\tilde{G}$ that does not converge; that is,

$\max N(\tilde{q}_o\tilde{\sigma}_1\tilde{\sigma}_2 \ldots \tilde{\sigma}_k) \not\to 1$, as $k \to \infty$.

The infinite sequence $\tilde{s} = \tilde{\sigma}_1\tilde{\sigma}_2\tilde{\sigma}_3 \ldots \tilde{\sigma}_k \ldots$ of $\tilde{G}$ must visit at least one state x of G_{obs} infinitely often. Let $u \in \Sigma^*$ be a string such that $\xi(x_o, u) = x$. By Lemma 1, we have $x = \text{convert}(\tilde{\delta}(\tilde{q}_o, \tilde{u}))$. The infinite sequence can be re-written as $\tilde{s} = \tilde{u}\tilde{\alpha}_1\tilde{\alpha}_2 \ldots \tilde{\alpha}_k \ldots$, where $\tilde{\alpha}_k \in \tilde{\Sigma}$. Because all circular strings strongly converge, there exists a nonblocking path

$$\{q_i\} \xrightarrow{\alpha_1} y_1 \xrightarrow{\alpha_2} \cdots y_{j-1} \xrightarrow{\alpha_j} y_j \cdots$$

such that $\{q_i\} = y_0 \subseteq x$, $y_j = \delta(y_{j-1}, \alpha_j)$, and $|y_0| = |y_1| = \cdots = |y_j| = \cdots = 1$. Denote by $p(y_j)$ the probability for $\tilde{G}$ to be in state y_j along the sequence $\tilde{s} = \tilde{u}\tilde{\alpha}_1\tilde{\alpha}_2 \ldots \tilde{\alpha}_k \ldots$. Then clearly $p(y_j) \leq p(y_{J+1})$. Furthermore, we can conclude that $p(y_j) \to 1$ as $j \to \infty$ because otherwise, there exists a different nonblocking path

$$\{q'_i\} \xrightarrow{\alpha_1} y'_1 \xrightarrow{\alpha_2} \ldots y'_{j-1} \xrightarrow{\alpha_j} y'_j \ldots$$

such that $\{q'_i\} = y'_0 \subseteq x$, $y'_j = \delta(y'_{j-1}, \alpha_j)$. From these two different infinite paths, we can obtain two different minimal and nonblocking realizations for some circular strings, which contradicts the assumption that all circular strings $v \in L_c(G_{obs})$ of the corresponding $G = \text{convert}(\tilde{G})$ and G_{obs} strongly converge. This proves the "IF" part.

(ONLY IF). Assume that G_{obs} has a circular string $v = \sigma_1\sigma_2 \ldots \sigma_j \in L_c(G_{obs}, x)$ that does not converge. Let $u \in \Sigma^*$ be a string such that $\xi(x_o, u) = x$. By Lemma 1, we have $x = \text{convert}(\tilde{\delta}(\tilde{q}_o, \tilde{u}))$. Let us prove that the infinite sequences $\tilde{s} = \tilde{u}\tilde{v}\tilde{v} \ldots \tilde{v} \ldots$ of $\tilde{G}$ do not converge by contradiction.

For a state $q \in x$, let $p^m(q)$ be the probability of $\tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta})$ being at q when $\tilde{s}$ visits x for the mth time. If $\tilde{s}$ converges, there must exist a unique state $q \in x$ such that $p^m(q) \to 1$ as $m \to \infty$. Let m be sufficiently large and ε is sufficiently small and $p^{m+1}(q) \geq p^m(q) \geq 1 - \varepsilon$. Consider

$$R(\{q\}, v) = (\{q\} \xrightarrow{\sigma_1} y_1 \xrightarrow{\sigma_2} \cdots y_{j-1} \xrightarrow{\sigma_j} y_j).$$

If $y_j \nsubseteq \{q\}$, then when the system starts at $\{q\}$ and after the occurrence of $v = \sigma_1\sigma_2 \ldots \sigma_j$, it may move outside of $\{q\}$, hence the probability of returning to $\{q\}$ after v is less, contradicting the assumption $p_{m+1}(q) \geq p_m(q)$. Therefore, $y_j \subseteq \{q\}$ and $R(\{q\}, v)$ is a realization. $R(\{q\}, v)$ is minimal because $\{q\}$ consists of only one state. $R(\{q\}, v)$ is nonblocking because the system is strongly detectable. Furthermore, $R(\{q\}, v)$ is the unique realization that is minimal and nonblocking, because otherwise, $q \in x$ is not the unique state such that $p_m(q) \to 1$ as $m \to \infty$. Finally, $|y_1| = \cdots = |y_j| = 1$ must be true because $\tilde{s}$ converges. Therefore, $v \in L_c(G_{obs}, x)$ is a convergent circular string, which contradicts the assumption that $v = \sigma_1\sigma_2 \ldots \sigma_j$ does not converge. This proves the "ONLY IF" part. □

Example 3 (*Continued*). Let us check circular strings in Fig. 3 and determine the detectability of the nephritis diagnosis system in Fig. 1. For the pair $(\{q_1, q_2\}, \beta\alpha)$, its transition graph $TG(x, v) = TG(\{q_1, q_2\}, \beta\alpha)$ is as in Fig. 4.

Clearly $R(\{q_1, q_2\}, \beta\alpha)$ is a minimal realization. Although it is nonblocking, it is not convergent. Therefore, the system is not strongly detectable.

For the pair $(\{q_1, q_2\}, \alpha)$, we have its transition graph $TG(x, v) = TG(\{q_1, q_2\}, \alpha)$ as in Fig. 5.

The minimal realization is $R(\{q_1\}, \alpha)$. It is not difficult to see that it is a convergent realization. Hence the system is detectable.

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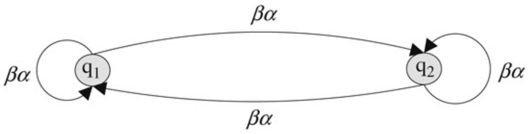

Fig. 4. The transition graph of the pair $(\{q_1, q_2\}, \beta\alpha)$.

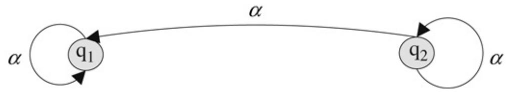

Fig. 5. The transition graph of the pair $(\{q_1, q_2\}, \alpha)$.

5. Conclusion

In this paper, we considered detectabilities of a probabilistic discrete event system. We defined (probabilistic) detectabilities both in a strong sense and in a weak sense. Compared with the detectabilities considered in our early work (Shu et al., 2006, 2007), the problem becomes much more complex because the transition probabilities must be taken into consideration. We defined the convergence for circular strings. Based on the definition of convergence for circular strings, we derived the necessary and sufficient conditions for (probabilistic) detectability and strong (probabilistic) detectability. All the results are illustrated by examples.

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