Design of Widely Tunable Semiconductor Lasers and the Concept of Binary Superimposed Gratings (BSG’s)

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Abstract—We propose a new concept of binary superimposed gratings (BSG’s) as multiwavelength grating reflectors and show it as an effective structural improvement over the existing designs, to both the fabrication process and device performance. We also present a study of key design issues for widely tunable lasers based on grating mirrors with a comb-like reflection spectrum and summarize simple design rules for the grating part of the laser, based on analytical and numerical analysis. The binary supergrating consists of elements of equal size whose refractive index is allowed to be one of two possible values, which are sequenced according to a binary optics formalism to effect a spatial superposition of multiple sets of single-frequency gratings, and its implementation is well within the standard e-beam lithography limits. The calculations of lasing frequency, mode, and side-mode suppression ratio of the tunable laser are formulated and presented along with numerical examples.

Index Terms—Distributed Bragg reflector lasers, gratings, laser tuning, wavelength division multiplexing.

I. INTRODUCTION

The emission wavelength of tunable semiconductor lasers based on distributed Bragg reflector (DBR) mirrors with a comb-like reflection spectrum have been shown to switch or discretely tune in a region about 100 nm [1], [2] and quasi-continuously tune in 62 nm [3], which is very desirable for wavelength-division multiplexing (WDM) systems of optical communication. The laser tunable in the 100-nm range, for instance, can provide around 120 wavelength channels separated by 100 GHz. In the most general case such a laser consists of four parts: two multiwavelength DBR mirrors, active (gain) region, and phase tuning region. The tunable DBR mirrors constitute a unique addition to the conventional Fabry–Perot (FP) laser that provides for a wide range and high side-mode suppression ratio. The phase tuning section is required for adjusting longitudinal modes. Consequently, there are a great many parameters determining the laser performance, and only a few of them are easily accessible and, when set in limited ranges, allow construction of a useful device. Empirical optimization of such a complicated laser looks hardly possible. A sizable amount of literature on theoretical description as well as experimental data of two types of DBR tunable lasers composed of either sampled gratings (SG’s) or superstructure gratings (SSG’s) have been built up over the last few years [1]–[16]. A few of them [4]–[6] present analysis of aspects of the laser operation and tuning mechanism. Very recently the idea of vernier tuning has been extended on distributed-feedback (DFB) lasers as well [17].

In this paper, we look into some of the unique and key design issues of widely tunable DBR lasers and consider the related general design guidelines for DFB’s. We also suggest a new DBR structure based on binary superimposed gratings (BSG’s) that seems more attractive than both of the existing options of SG’s and SSG’s because it is easier to implement and has a built-in degree of freedom in adjusting the individual zero-bias positions and amplitudes of the reflectance peaks. This adjustability could extend the continuous tuning range for the same number of reflectance peaks in the same waveguide because it allows optimization of the individual peak spacings to account for wavelength-dependent, current-induced index change and the structure and current-dependent gain spectrum. We analyze the side-mode suppression ratio (SMSR), which is an important figure of merit in controlling possible noise in WDM optical communication systems. The SMSR in lasers based on tunable DBR’s with multiple reflection peaks depends heavily on the overlap or partial overlap of peaks other than the main pair and is affected by such factors as limited DBR length, internal waveguide losses, and also the number and positions of the peaks. At last, we discuss the perturbations of the lasing frequency originating from carrier-induced index change in the gain cavity of the laser, a consequence of threshold current changes while tuning.

II. DESIGN ISSUES AND RULES

The normal operation and design of the widely tunable laser [7] is based on the principle of the Vernier and is illustrated in Fig. 1. The laser consists of front and back tunable DBR reflectors, a gain section, and a phase tuning section. The reflectivity spectrum (magnitude) of the front reflector, $R_f(\lambda)$, consists of an array of peaks separated by $\Delta \lambda_f$. Current $I_f$ driven through the front reflector can produce a maximum spectrum shift of $D_f \geq \Delta \lambda_f$. The reflectivity spectrum of the
Fig. 1. The operational principle of widely tunable laser with comb-like reflectors. (a) Schematic view of the widely tunable DBR laser. (b) Spectra of front and back reflectors, their shift with controlling current, and longitudinal cavity modes of the laser.

back reflector, \( R_b(\lambda) \), contains an array of peaks separated by a slightly different spectral spacing, \( \Delta \lambda_b \). The back reflector spectrum can be shifted by \( D_b \geq \Delta \lambda_b \) while a current \( I_b \) is supplied. The total number of the reflection peaks in each spectrum and the separations \( \Delta \lambda_f \) and \( \Delta \lambda_b \) are arranged so that not more than one pair of peaks may coincide at any combination of currents \( I_f \) and \( I_b \). In order to obtain lasing at a given wavelength \( \lambda_L \) one needs: 1) to install the current \( I_f \) to set one of the reflectance peaks of \( R_f(\lambda, I_f) \) at \( \lambda = \lambda_L \), and suppress it at the adjacent unintended peaks of total reflection function \( R_f(\lambda) \). This condition will certainly be satisfied if the difference in peak spacing is greater than the width of the peaks; \( \Delta \lambda_f > W \), where \( W \) is the largest expected width of an individual reflection peak. In practice, however, \( \Delta \lambda_f \) is allowed to be just a fraction of \( W \), say \( W^* \), depending on the desirable SMSR (for an estimate of \( W^* \) see Section IV, and [6] for the case of SG):

\[
\Delta \lambda_f > W^*. \tag{1}
\]

It is self-evident that narrow reflectance peaks (of long and weak gratings, as a DBR reflectance peak width is defined by the grating coupling constant, \( \kappa \)) are desirable for large SMSR. In reality, the grating length is restricted by the waveguide losses if not by other factors. The requirement that the effective coupling coefficient of the supergrating be at least of the order of the losses gives an estimation of the peak width which results in

\[
W \geq (\lambda^2 \alpha)/2n \tag{2}
\]

where \( n \) is an effective index. Another restriction for \( \Delta \lambda_f \) also comes from the requirement of single frequency generation, as graphically illustrated in Fig. 2

\[
(N - 1)\Delta \lambda_f - (N - 2)\Delta \lambda_f > W^* \tag{3}
\]

where \( N \) is a number of reflectance peaks in each reflector spectrum. The last relation could be considered as a restriction for the number of peaks or \( \Delta \lambda_f \)

\[
N < 2 + (\Delta \lambda_b - W^*)/\Delta \lambda_f. \tag{4a}
\]

\[
\Delta \lambda_f < (\Delta \lambda_b - W^*)/(N - 2). \tag{4b}
\]

Equations (1) and (4b) then represent the lower and upper bounds for \( \Delta \lambda_f \). The tuning range is

\[
\lambda_{end} - \lambda_{beg} = (\Delta \lambda_f + \Delta \lambda_b)(N - 1)/2. \tag{5}
\]

As an example, we will consider below an InGaAsP-based laser emitting around \( \lambda = 1.55 \) \( \mu \)m. Let the current induced tuning range of an individual peak be \( \Delta \lambda = \lambda |\Delta n|/n \approx 10 \) nm, and the waveguide losses including the tuning current induced losses be 40 cm\(^{-1} \) (large change in absorption losses must necessarily accompany a large change in refractive
From (2), the minimal peak width $W$ is 1.4 nm and it follows from (1) that the minimal difference in peak spacing $\Delta \lambda_d$ is around $W^* = 1$ nm. So $\Delta \lambda_f$ and $\Delta \lambda_d$ are taken to be 10 and 9 nm, respectively. From (4) we find the maximum possible number of reflection peaks $N = 10$ for a tuning range 95 nm. Therefore, to realize such a laser, one needs a supergrating with 10 reflection peaks. In order to have a reasonably high reflection, the effective coupling constant for each peak should be at least of the order of the losses which are $40 \text{ cm}^{-1}$.

III. MULTIWAVELENGTH GRATING DESIGNS

A. Sampled Gratings and Superstructure Gratings

As already reflected in the preceding sections, an FP laser is made tunable by the addition of tunable DBR grating mirrors with a comb-like reflection spectrum. Direct superposition of holographic exposure would be, perhaps, the simplest way to create the multiwavelength reflectors [18]–[21], and such gratings could be used to demonstrate the Vernier tuning [22]. However, an analog superposition does not seem to be a promising approach in the case of semiconductor waveguides due to high nonlinearity of the exposure, resist development, and etching of semiconductor layers. Up to now, only two types of gratings were proposed and demonstrated for the multiwavelength DBR mirrors that are free from the shortcomings of analog superimposed gratings: the sampled gratings (SG’s) and the superstructure gratings (SSG’s). The following is a brief review and comparison of these methods of achieving multiwavelength gratings.

The idea of SG’s relies on periodical modulation (sampling) of the grating such that zones of the grating are periodically removed [Fig. 3(a)]. It is the period of modulation that determines the spectral separation between peaks. For small duty cycles, the number $N$ of peaks in reflectivity spectrum is approximately equal to the inverse of the duty cycle [6]. Thus, a large number of peaks requires a small duty cycle. For the same reason, the amplitudes of peaks in the Fourier spectrum of SG’s are inversely proportional to the number of peaks [5]. For given peak width and amplitude, such “sampling” translates into a linear increase in length with the number of peaks. Assuming a reasonably high reflectivity of 0.6 and denoting a coupling coefficient for the unremoved part of the grating $k_0$, the number of reflection peaks $N$ can be estimated from the relation

$$\left(\frac{k_0}{N}\right)L \approx 1.$$  \hspace{1cm} (6)

The total length of any grating $L$ is limited by absorption. According to [5], a current density of 10 kA/cm$^2$ injection into InGaAsP($\lambda_g = 1.3 \text{ \mu m}$)-InP-based grating waveguides yields about 8-nm reflection wavelength tuning in the 1.55-\mu m region, but it also increases the internal losses from 8 to 50 cm$^{-1}$. Even for rather large values of $k_0 = 150 \text{ cm}^{-1}$ and $L = 500 \text{ \mu m}$ the number of reflection peaks is about 7–8. Usage of a small duty cycle grating will result in small reflection peak amplitude, and larger duty cycle will lead to narrowing of the overall reflection spectrum envelope and reduction of the number of peaks. The Fourier spectrum envelope of SG’s shapes like $\sin(x)/x$, and there are no free parameters to...
change the spectrum shape other than scaling. It means that reflection peaks are intrinsically of different amplitudes, and lasing in different bands of the range of tuning will have very different thresholds and efficiencies.

The idea of SSG is a periodical modulation of the grating period [Fig. 3(b)] or, equivalently, of the grating phase. Also, such kinds of gratings can be called periodically chirped gratings. The spectral separation between channels, as in the case of the SG, is determined by the modulation period. Actual design implementation of the idea consists in dividing the modulation period into a set of subsections and filling each subsection by a grating of constant pitch width and depth, but with the grating phase varying from subsection to subsection [Fig. 3(c)]. In contrast to SG, SSG does not contain grating-free zones. For that reason, the amplitudes of peaks in the Fourier spectrum are bigger than that of SG for the same length. As noted in [5], they decay with the number of channels as $1/\sqrt{N}$. Using the same criteria as in (6), the number of channels available with SSG may be found approximately from the equation

$$\left(\kappa_0/\sqrt{N}\right)L \approx 1.$$  \hspace{1cm} (7)

Numerical adjustment of the grating phases (or pitch width) of the subsections makes it possible to construct a SSG with highly uniform amplitude of reflection peaks. A normalized average squared nonuniformity of $9 \times 10^{-3}$ was calculated for $N = 11$ SSG’s. However, technological implementation of the SSG of desirable uniformity requires extremely precise lithography to reproduce the designed ultrafine grating phase shifts of each section correctly. Not surprisingly, the measured transmittance spectrum, as shown in [5], has a nonuniformity of several decibels that corresponds to reflectance peaks nonuniformity of several percents.

In order to understand the intrinsic technological demands in making such a SSG, let us estimate the value of a linear grating shift, $d_{\text{min}} = \psi/\Delta\lambda_0/2\pi$, corresponding to the phase difference between neighboring subsections. Let us denote $\lambda_{\text{end}} = \lambda_{\text{end}}/2n$ and $\lambda_{\text{seg}} = \lambda_{\text{seg}}/2n$ the maximum and minimum values of the grating pitch width, respectively, where $\lambda_{\text{end}} \approx 1.6 \mu m$ and $\lambda_{\text{seg}} \approx 1.5 \mu m$ is an optimistic estimation of the spectral boundaries of the reflection spectrum envelope, and $n \approx 3.2$ is the waveguide modal index. In the region of wavelengths around $\lambda_0 \approx 1.55 \mu m$, the desirable spectral spacing between adjacent reflection peaks ($\Delta \lambda = 10 \text{ nm}$) and the equivalent waveguide index, taking into account the effective index dispersion ($\kappa_{\text{seg}} = n - \lambda_0 \Delta\eta/d\lambda \approx 3.6$), determine the period of the grating modulation $\Delta_s = \lambda_0^2/2n\kappa_{\text{seg}} \Delta\lambda \approx 33 \mu m$. To achieve a linear change in the grating period one needs a parabolic distribution of the subsection phase with respect to the section position. The necessary phase shift between neighboring sections is equal to

$$d_{\text{min}} = \frac{\Delta\lambda/\Delta_0}{\lambda_s/\Delta_0^2} = \frac{\Delta_s}{M_s^2} \hspace{1cm} (8)$$

in units of length, where $M_s$ is a number of sections in one period, $\Delta = |\lambda_{\text{end}} - \lambda_{\text{seg}}|/\lambda_0 = (\Delta\lambda_{\text{end}} + \Delta\lambda_{\text{seg}})/2$. The authors of [5] used $M_s = 50$ in theoretical calculations to demonstrate a near perfect uniformity of reflection peaks, and $M_s = 18–25$ in experiments that did not yield the computed uniformity. Although the phase distribution can be altered somewhat by a numerical optimization procedure, the above estimation of $d_{\text{min}} \approx 1 \text{ nm}$ at $M_s = 50$ gives the order of magnitude of the required precision in fabricating the SSG gratings of a uniform (or square shaped) peak envelope, a tough demand to meet.

### B. The Concept of Binary Superimposed Gratings (BSG’s)

Although the principle of SSG is sound for application in broad tunable lasers, another design of a comb-like spectrum reflector might be equally effective but much less demanding in actual implementation. The superimposed grating design we describe here is a binary extension of the same concept covered in [18]–[21] of the analog form, where multiple sets of Bragg gratings are superimposed on top of each other in the same volume or in a multiple layers [21] staged together. But, for implementation in semiconductor devices, it is probably the easiest to adopt the concepts of binary optics and create a binary form of the superimposed gratings, or BSG. BSG’s consist of an array of segments of equal length $s$. The effective waveguide index of each segment $n_i(\lambda)$ is allowed to be changed between two values, e.g., by $\Delta n \cdot (m - 1/2)$, with $m = 1$ or $m = 0$

$$n_i(\lambda) = \begin{cases} n(\lambda) + \Delta n/2, & \text{if } i \text{ is odd} \\ n(\lambda) - \Delta n/2, & \text{if } i \text{ is even} \end{cases} \hspace{1cm} (9)$$

where $i = 1, 2, 3, \ldots$ is the segment number (or digital position), $n(\lambda)$ is a waveguide effective index including both waveguide and material dispersion, and the function $f(x)$ is just a weighted sum of sinusoidal functions of desirable period $\Delta_j$.

$$f(x) = \sum_{j=1}^{N} a_j \sin \left( \frac{2\pi x}{\Delta_j} + \Psi_j \right). \hspace{1cm} (10)$$

The periods $\Delta_j$ relate to the spectral positions of reflection peaks $\lambda_j$ through the waveguide index $n(\lambda)$.

$$\Delta_j = \lambda_j/2n(\lambda_j). \hspace{1cm} (11)$$

So, the proposed BSG is nothing but an array of gratings (10) superimposed analogly and then subjected to binary digitization according (9). The digitization, of course, means a distortion to the analog supergrating in performance. The phases $\Psi_j$ are free parameters that could be used for optimization of the BSG reflection spectrum. The factors $a_j$ were found to be varied to obtain a desirable relation between the reflection peaks. This could be done by a traditional numerical optimization scheme, or empirical adjusting when $N$ is small, or perhaps by more elegant analytical formulations, which is beyond the scope of this paper.

Once the design is completed, the binary grating would look like a single-depth grating of a period $p \cdot s$, where $p$ is an integer number, $p = \text{round}(\Delta_0/s)$, and a phase shift of an integer number of $(2\pi/\Delta_0)s$ is introduced where required in accordance with (9) and (10). The profile of the BSG is schematically illustrated in Fig. 3(d). The phase shift distribution along the grating length may be very similar to that
of an SSG [Fig. 3(c)] if we create the BSG assuming \( \Psi_j = 0(j = 1, \ldots, N) \) and equidistant reflectance peaks positions \( \lambda_j \). However, assuming nonzero \( \Psi_j \), one can obtain grating phase distributions which have nothing common with that of the SSG, while their reflectance spectra remain very similar. We have found also that a shape of the reflectance spectrum envelope of the SSG-like BSG is sensitive to waveguide losses, while in the case of the BSG created with random \( \Psi_j \) the envelope remains flat with the losses change [23]—a property which can be important for applications.

One advantage of the proposed BSG reflector is its binary nature (equal size base elements of two index values) that should allow for easier and more reproducible implementation by a standard electron beam lithography. Another advantage of the BSG is a freedom in the peaks’ positions. Really, the algorithm described above does not imply any specific relations between \( \lambda_j \). In Section III-C, while discussing modifications of the simple design rules resulting from the dispersion of the current-induced waveguide index change, we will show how important this freedom is for tunable lasers and give an example of the BSG with irregular peaks’ positions.

In order to understand how the idea of the BSG works, let us first consider a case of a single reflectance peak, \( N = 1 \). If each half-period of the grating contains an integer number of segment (so, \( s = \Lambda/2, \Lambda/4, \ldots, \Lambda/8, \ldots \)), the resultant binary grating will be just a periodic sequence of rectangular shaped pitches with a well-known reflection spectrum. The first-order (\( \Delta = \Lambda/2n \)) coupling strength equals to \( \kappa_0 = 2\Delta n/\lambda \). The question is what will happen if the grating period cannot be made of an integer number of segments, e.g., if the desired period is not exactly a multiple of an exposure element size of an e-beam machine? How will the reflection peak amplitude, width, and position depend on the segment size in this case?

In the case of periodic index profile, the coupling constant is proportional to Fourier component

\[
\kappa = \frac{4\pi}{\lambda} \int_0^\Lambda n(x) \sin\left(\frac{2\pi x}{\Lambda}\right) dx
\]

(for the sake of simplicity we omitted a cosine term in the Fourier integral, that is, we suppose a particular phase of the grating profile). In the case of quasi-periodic index profile, such as the binary gratings prepared according to (9)–(11), integrating over the period should be replaced by integrating over the total grating length \( (1/\Lambda) \int_0^\Lambda dx \to (1/L) \int_0^L dx \). As a result, we have

\[
\kappa_s = \frac{4\pi}{\Lambda} \sum_{i=1}^{M} \frac{\Delta n}{2} \text{sign}\left\{ \sin\left[\frac{2\pi s(\Lambda - 1/2)}{\Lambda}\right]\right\}
\]

\[
\cdot \int_{s(i-1)}^{s i} \sin\left(\frac{2\pi x}{\Lambda}\right) dx
\]

\[
= \frac{\Lambda \Delta n}{L} \sin\left(\frac{\pi s}{\Lambda}\right) \sum_{i=1}^{M} \text{sign}\left\{ \sin\left[\frac{2\pi s(\Lambda - 1/2)}{\Lambda}\right]\right\}
\]

\[
\cdot \sin\left(\frac{2\pi s(\Lambda - 1/2)}{\Lambda}\right)
\]

where \( M \) is the number of segments composing the grating. To estimate the sum in the last equation one can use periodicity of sine function and shift all points \( s(i-1/2), i = 1, 2, \ldots, M \), to the first period \([0, \Lambda]\). Assuming then their uniform distribution within the first period, we can substitute \( \sum_{i=1}^{M} (\cdots) \to (M/\Lambda) \int_0^\Lambda \sin^2(2\pi x/\Lambda) dx \) and obtain a useful and simple approximation

\[
\kappa_s \approx \frac{\sin(\pi s/\Lambda)}{\pi s/\Lambda},
\]

This derivation for the coupling constant of a binary grating is to a certain degree analogous to an analysis of signal transformation in the sampling theory of communication (see, for example, [24]).

Equation (14) was confirmed numerically by calculating the reflectance spectra from a binary grating created as described above, for varying the segment size \( s \). Then the coupling strength of the binary grating was determined from the maximum reflectance \( R \) as \( \kappa_R = (1/2L) \ln\left[1 + (1 + \sqrt{R})/(1 - \sqrt{R})\right] \), which follows from the well-known expression of the maximum reflectance of a lossless grating \( R = \tan^2(\kappa L) \) [25]. The results of the numerical procedure are plotted in Fig. 4. They are in perfect agreement with (14) everywhere except at some singularities, where numerically calculated plot \( \kappa(s) \) contains very narrow, \( \approx 0.01\text{nm} \) (see inset on Fig. 4) peaks at \( s = LP/Q \) (\( P \) and \( Q \) are small integer numbers). These singularities exist also in an accurate analytical analysis of the BSG’s coupling strength [26]. However, they are hardly of any practical importance as it is inevitable to have some deviations, however small, in writing the segments that spoils the exact division of \( s = LP/Q \). Thus, (14) shows us the cost of adopting the concept of binary grating in terms of coupling strength. Note that the peak width and position is negligibly affected by the segment size (Fig. 5).

Coupling strength in the case of \( N > 1 \) can be estimated on the basis of (14) following the approach of [5] by introducing a multiplication factor \( 1/\sqrt{N} \) resulting from the Parseval’s theorem. In other words, one can use the following analytical estimation for the coupling strength of BSG:

\[
\kappa_s(N) = \frac{1}{\sqrt{N}} \frac{2\Delta n \sin(\pi s/\Lambda)}{\pi s/\Lambda}
\]

instead of involving a full numerical computation.

Now, returning to the BSG design guided by (9)–(11), we compute the reflectivity spectrum of an example BSG by the transfer matrix method (Fig. 6). For this example case we took the segment size \( s = 0.05 \text{\mu m} \), the grating length \( L = 500 \text{\mu m} \), and the effective index change \( \Delta n = 8 \times 10^{-3} \). We designed the binary grating for \( N = 11 \) reflection peaks at wavelengths of \( \lambda_1 = 1500 \text{nm}, \lambda_2 = 1510 \text{nm}, \ldots, \lambda_{11} = 1600 \text{nm} \). The waveguide index dispersion was \( n = 3.2 - 2.6 \times 10^{-4} \times (\lambda [\text{nm}] - 1550) \). In the case of lossless waveguides, the calculated nonuniformity \( \delta_R \) of reflectance peaks amplitudes \( R_l, l = 1, \ldots, N \)

\[
\delta_R = \sqrt{\langle (R_l - \langle R_l \rangle)^2 \rangle / \langle R_l \rangle^2}
\]
was found to be $6 \times 10^{-4}$ after coarse optimization of coefficients $\alpha_j$ (the brackets $\langle \ldots \rangle$ mean averaging, e.g., $\langle R_j \rangle = (1/N) \sum_{i=1}^{N} R_i$). The width $W$ of an individual peak is 1 nm.

Compared with the SSG approach, the BSG has the advantage in implementation, as exhibited in the above example where the used segment size many times (50–100) larger than the $d_{\text{min}}$ of SSG [see, e.g., (8)] yields comparable, if not superior, performance. Moreover, its reflectance characteristics do not degrade much even if we double the segment size. Shown in Fig. 7 are coupling coefficient versus segment size determined in accordance with (15) (open symbols) and estimated from the numerically calculated reflectance spectra (filled symbols).

It is also noteworthy that the numerical calculation results in 10%–12% less coupling strength than the analytical estimation (15) for number of superimposed gratings ranging from 2 to 11 (see inset on Fig. 7). Physically it means that a part of the coupling strength is distributed to the several small reflection peaks which exist outside the region of the $N = 11$ peaks in Fig. 6.

C. Beyond the Simple Design Rules

An additional advantage of BSG lies in that it offers the freedom in adjusting the individual position of the reflectance
Fig. 6. Example of the reflectance spectrum of the BSG.

Fig. 7. The reflectance spectrum of the BSG’s (filled circles), the coupling coefficient estimated from the reflectance spectrum (filled squares), and the coupling coefficient calculated analytically (15) versus the segment size, for the lossless $N = 11$ BSG of $L = 500\,\mu$m length. The inset shows the coupling coefficients calculated numerically (filled diamonds) and analytically (open diamonds) versus the number of superimposed gratings, assuming the segment size $s = 50\,nm$ and $L = 500\,\mu m$.

peaks, which is absent in using an SG or SSG. Thus, this advantage translates into one of improved performance in the presence of wavelength-dependent index, loss, and gain. Accurately analyzing the spectra of SG and SSG one can find that the reflectance peaks actually are not equally spaced in the wavelength domain. Instead, they are equally spaced in a scale of $n(\lambda)/\lambda$. It means that the wavelength-scale peak spacing around $\lambda_{\text{seg}}$ is smaller than that of $\lambda_{\text{endl}}$. For $\lambda_{\text{seg}} = 1.5\,\mu m$ and $\lambda_{\text{endl}} = 1.6\,\mu m$ this difference is about $\delta_1 = 14\%$.

In fact, it is not necessary to use exactly equidistant peaks. Tuning on the base of the Vernier concept will work just fine if using any monotonic function $\lambda^* = g(\lambda)$. Moreover, the current-induced index change is stronger closer to bandgap. For InGaAsP material with $\lambda_g = 1.3–1.4\,\mu m$, this difference at $\lambda_{\text{seg}} = 1.5\,\mu m$ and $\lambda_{\text{endl}} = 1.6\,\mu m$ can be of the order of $\delta_2 = 50\%$. Therefore, an appropriate correction of peak positions should be done, and could be done in the BSG scheme, for optimal utilization of tuning properties of the
waveguide. As illustrated in Fig. 8, in the case of the SG and SSG some wavelength regions within the continuously tunable range are covered twice. Using the BSG, we can avoid (or minimize) the duplicated covering thereby effectively expanding the tuning range or reducing the current demand. For the numbers cited above, the extension of the tuning range would be 30%. To best utilize this freedom, detailed knowledge of the current-induced refractive index change, specific to the chosen structure and material, is required. Examples of the BSG with the peak spacing varying linearly from 0.8 to 2 nm are shown in the bottom of Fig. 8. We show also the freedom in individually adjusting the peaks’ amplitudes. The relative amplitudes of the peaks in the last example are chosen to be equal to 1 or 1/2 which implies no practical goal, but demonstrates the design freedom only.

IV. SIDE-MODE SUPPRESSION RATIO (SMSR)

Side modes can cause unwanted signal crosstalk in WDM optical communication systems. Because of the multiple-peak nature of the proposed BSG reflectors, the SMSR is an especially important issue for this class of tunable lasers. Below we examine an expression for SMSR and its specific implication on BSG structure parameters and evaluation.

#### 17a. Loss-Difference Conditions

In evaluating the single-mode performance of the tunable lasers (SMSR), the ratio of the power of the main mode to that of the highest side mode can be estimated in terms of the loss difference between the primary mode and the next highest mode (see, for example, [27]), and is given approximately by

\[
\text{SMSR} = \frac{r_j \ln(r_j^{-1}2L)}{(\text{R})_\text{esp} (1-r_j)^2 v_g \{\Delta \alpha - \Delta g\} P}
\]

where \(\Delta \alpha\) is the total loss difference between the first and second strongest mode and is equal to the sum of the mirror loss difference \(\Delta \alpha_{\text{m}}\) and the internal loss difference \(\Delta \alpha_{\text{i}}\); \(\Delta g\) is the modal gain difference, \(\Gamma\) is the confinement factor, \(g_{\text{th}}\) is the threshold gain, \(P\) is the output power, \(k\) is the photon energy, \(r_j\) is the mirror reflectance of the second strongest mode, \(v_g\) is the group velocity, \(n_{\text{esp}}\) is the population inversion factor defined as

\[
n_{\text{esp}} = \left[1 - \exp \left(\frac{\eta_{\text{m}} - E_f}{kBT}\right)\right]^{-1}
\]

where \(E_f\) is the energy separation between the quasi-Fermi levels, and \(T\) is the temperature. For the minimum value of SMSR, and if the worst case is assumed, the numerator in the brackets is at least \(|\Delta \alpha_{\text{m}}|_{\text{min}} - |\Delta (\Gamma g_{\text{th}} - \alpha_{\text{i}})|_{\text{max}}\) where the subscripts indicate minimum and maximum anticipated values. Equation (17a) can thus be rewritten

\[
|\Delta \alpha_{\text{m}}|_{\text{min}} = \frac{\Gamma g_{\text{th}}(\omega_{\text{esp}}) (1-r_j)^2 v_g \text{SMSR}}{r_j \ln(r_j^{-1}2L) P}
\]

\[
+ |\Delta (\Gamma g_{\text{th}} - \alpha_{\text{i}})|_{\text{max}}.
\]

\(|\Delta \alpha_{\text{m}}|_{\text{min}}\) is a key factor when estimating the minimum value of \(\Delta \lambda_{\text{m}}\) in design required to arrive at a given side-mode suppression. Note that a value for \(r_j\) must be presumed in advance, but small variations in this variable were not found to greatly affect the prefactor. Once known from (17b), the minimum mirror loss difference can be related to the reflectivities, resulting in the condition

\[
\frac{1}{2L_\alpha} \left\{\ln \frac{1}{R_f(\lambda_{\alpha}) R_b(\lambda_{\alpha})} - \ln \frac{1}{R_f(\lambda_{\alpha}) R_b(\lambda_{\alpha})}\right\} 
\geq |\Delta \alpha_{\text{m}}|_{\text{min}}
\]

(18)

where \(R_f(\lambda) R_b(\lambda)\) is the product of the front and back BSG reflectivities, \(\lambda_{\alpha}\) is the wavelength of the primary peak of \(R_f(\lambda) R_b(\lambda)\), and \(\lambda_{\alpha}\) is the wavelength of the adjacent peak of the product. If the peak widths of the front and back BSG’s are the same, then \(\lambda_{\alpha} = (\lambda_f + \lambda_b)/2\) where \(\lambda_f\) and \(\lambda_b\) are the resonant wavelengths of the reflectivity peak of the front and back BSG which are adjacent to \(\lambda_{\alpha}\). Noting that \(\lambda_{\alpha}\) is \((\lambda_f - \lambda_b)/2 = \Delta \lambda_{\alpha}/2\) away from either of the individual peaks, (18) can be rewritten in terms of the function \(R(\lambda - \lambda_{\alpha})\) describing reflectivity of an individual peak:

\[
\frac{R(\Delta \lambda_{\alpha}/2)}{R(0)} \leq \exp(-|\Delta \alpha|_{\text{min}}L_\alpha)
\]

(19)

where it is assumed that the magnitude of the reflectivity peaks for both BSG’s are the same for simplicity. The minimum peak
Fig. 9. The calculated zero-bias reflectance spectrum of the binary superimposed gratings optimized for an internal loss of 8 cm\(^{-1}\). The upper graph is an overlay of the reflectance spectrums of the individual gratings, the lower graph the product of the two spectra.

spacing difference for the gratings could be thus derived if the peak shape function \(R(\lambda)\) is known.

For a tunable laser structure with \(\Delta \lambda = 0.8\) nm and active region length \(L_a = 350\) \(\mu\)m, and other parameters given in Table I, numerical calculations of the BSG reflectance peaks (Fig. 9) yield almost equal LHS and RHS of (19). For a first-order estimation, one can assume a Gaussian shape of \(R(\lambda)\) of width \(W_{\text{FWHM}}\), yielding from (19)

\[
\Delta \lambda_f \geq W_{\text{FWHM}} \sqrt{\frac{\Delta \omega_{\text{m}} L_a}{\hbar}} \tag{20}
\]

which is an explicit requirement on the peak spacing difference. Comparing to (1), one can find in this case \(W^* \approx 0.7 \cdot W_{\text{FWHM}}\).

### B. Numerical Calculations

In addition to analytical estimation of SMSR given above, we present a numerical simulation of a particular example structure. To predict the SMSR for a BSG laser, the focus of the calculation is to find the values of \(\Delta \omega_{\text{m}}\) by finding the modes of the laser, and then applying the standard analytical approximation for SMSR (17). The worst-case values of the other parameters in (17) as well as material parameters are listed in Table II. The values of \(\Delta \omega_{\text{m}}\) were evaluated for coarse tuning, i.e., current was applied to one BSG to match the other on one of the resonant wavelengths. Sourcing current on both BSG’s, which is necessary for continuous tuning, results in lower combined reflectivity due to increased losses in the other grating; the decreased SMSR under these less favorable conditions will also be estimated below. The value of current required to achieve such matching in each case was found by equating the resonant wavelengths corresponding to the particular grating period component of the two BSG’s (\(\Lambda_j\’s\)), accounting for refractive index varying with the root of the current and linearly with wavelength (with nominal index \(n_0\) at wavelength \(\lambda_0\))

\[
J_f = \left[ \left( \frac{\partial n}{\partial \sqrt{J}} \right)^{-1} \left( \frac{\Lambda_j}{\Lambda_f} - 1 \right) \right] \left\{ n_0 + \left( \frac{\partial n}{\partial \lambda} \right) (\lambda_j - \lambda_0) \right\}^2.
\]

The superscripts pertain to the back and front gratings, and the above equation assumes that current is applied to the front grating.

The magnitude and phase of the reflection spectra of both gratings were obtained using the transfer matrix method accounting for dispersion and additional internal loss variation in the BSG’s with current. Longitudinal mode frequencies in the neighborhood of the various reflectivity product peaks were found from the phase condition

\[
2\beta_a L_a + 2\beta_p L_p + \phi_f + \phi_b = 2m\pi \tag{22}
\]

where \(\beta_a = 2\pi n_a / \lambda\), \(\beta_p = 2\pi n_p / \lambda\) are the propagation constants in the active and phase regions and \(\phi_f\) and \(\phi_b\) are the phases of the reflectivities of the back and front BSG’s. The second term is adjusted such that the principle mode is always at a reflectivity maximum, which in the actual device is accomplished by varying the current through the phase tuning section. Once the wavelengths of modes are found, \(\Delta \omega_{\text{m}}\) is obtained from the difference between the first and second highest reflectivity products. The effects of refractive index variations in the active region due to varying threshold gain were not accounted for in this numerical analysis but can be easily included, as shown in Section V, if a gain function is known.

The peak wavelengths for coarse tuning, as found from mirror loss alone, are given in Fig. 10 as a function of the
current density. Fig. 11 shows the computed wavelengths and reflectivity product of the modes when current specified by (21) is applied either on the left or right BSG. The general trend is that as the primary wavelength is tuned away from the zero-bias value, the peak reflectivity product decreases due to increased current-induced losses in the grating. At the longer wavelength end of the tuning range, however, a slight increase of reflectivity is observed, which is attributed to nonuniformity already existing in the nonbiased gratings of Fig. 9. While the reflectivity at the main modes decreases with increased grating bias, the side modes do not follow this general trend (compare, for instance, the side-mode reflectivity at 0 and 1.46 kA/cm). Therefore, and also because of the increase in the threshold gain, the SMSR is expected to decrease as the primary wavelengths become further from their zero-current values. The worst case here is seen at the 1.51-μm wavelength, and the corresponding mirror loss difference between the primary and next highest mode is 17.4 cm. Note also that for wavelengths near the extremities of the tuning regions significant side modes appear at the opposite end of the tuning range, which highlights the importance of (4) in Section II.

Both the result above and the laser structure data provided in Table I are used in (17a) for SMSR. With the structure considered here, the first factor before the brackets turns out to be more than 2 \times 10^4 (mW). The last parameter needed is the threshold gain, which is given by

\[ (\Gamma g_{th})_{max} = \alpha_0 + \frac{1}{2\bar{n}_a} \ln \left( \frac{1}{R_f R_b} \right) + (\alpha_{np})_{max} \frac{L_p}{L_a}. \]  

The calculated minimum mode suppression ratio at 3 mW for this laser while coarse tuning is 40 dB. The SMSR in fine tuning can also be estimated by realizing that the maximum spectral shift of one BSG required for coarse tuning is similar to the maximum shifts (and hence currents) needed for both BSG’s in fine tuning, which is approximately the peak spacing. From comparing the maximum reflectivity product peaks of Fig. 11 for zero bias, where \([R_f(\lambda)R_b(\lambda)]_{max} = R_{max}^2\), and high bias, where \(R_f(\lambda)R_b(\lambda) = R_{max}(R_{max} - \Delta R)\), the reflectivity product peak for fine tuning \((R_{max} - \Delta R)^2\) can be solved for. With the conservative assumption that the main peak decreases due to increased losses in the manner described above, while side peaks are the same as in coarse

---

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0)</td>
<td>loss in active region</td>
</tr>
<tr>
<td>((\alpha_{np})_{max})</td>
<td>maximum anticipated loss in phase tuning section</td>
</tr>
<tr>
<td>(L_a)</td>
<td>length of active region</td>
</tr>
<tr>
<td>(L_p)</td>
<td>length of phase region</td>
</tr>
<tr>
<td>(\Delta [\Gamma g - \alpha]_{max})</td>
<td>maximum difference in modal gain between primary and secondary mode</td>
</tr>
<tr>
<td>(P)</td>
<td>optical output power for above parameter</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_m)</td>
<td>middle peak</td>
</tr>
<tr>
<td>(\Delta \lambda_r)</td>
<td>peak spacing of right BSG</td>
</tr>
<tr>
<td>(\Delta \lambda_l)</td>
<td>peak spacing of left BSG</td>
</tr>
<tr>
<td>(N)</td>
<td>number of peaks</td>
</tr>
<tr>
<td>(n_0)</td>
<td>median refractive index</td>
</tr>
<tr>
<td>(\delta n)</td>
<td>refractive index step about (n_0)</td>
</tr>
<tr>
<td>(s)</td>
<td>minimum length of constant-index section</td>
</tr>
<tr>
<td>(L_{BSG})</td>
<td>BSG length</td>
</tr>
<tr>
<td>(\partial n/\partial \lambda)</td>
<td>refractive index change with wavelength</td>
</tr>
<tr>
<td>(\alpha_{BSG})</td>
<td>zero-bias BSG internal loss</td>
</tr>
<tr>
<td>(\partial n/\partial (1/\lambda))</td>
<td>refractive index change with root of current density</td>
</tr>
<tr>
<td>(\partial \alpha_{BSG}/\partial (1/\lambda))</td>
<td>internal loss change with root of current density</td>
</tr>
</tbody>
</table>
Fig. 10. Peak wavelength versus current density sourced on one of the BSG's. The negative sign refers to sourcing current on the back grating, while positive current means the front grating (larger $\Delta \lambda$).

Fig. 11. Reflectivity product peaks of modes for various currents. For quicker computation, only those modes in a 2-nm radii about each of the five highest reflectivity peaks were looked for, so low-reflectivity modes between the peaks were omitted. $J_L$ and $J_R$ correspond to sourcing current on the back and front gratings, respectively.

tuning, the worst-case SMSR for this particular structure during continuous tuning is estimated to be 36 dB at 3 mW.

The above calculations could extend to include comprehensive simulations of laser behavior, including $L-I$ characteristics under different tuning currents. However, the scope of this paper was not to determine the threshold current and slope efficiency, which can be done by standard methods once the losses associated with given tuning currents are known (i.e., the simulation involved would not be unique to BSG). Nevertheless, the current injected into the active region, which depends on the primary mode, also has an effect on the lasing frequency itself, the subject of the next section.

V. SELF-CONSISTENT DERIVATION OF LASING FREQUENCIES AND MODES

In the previous section, the cavity modes were simulated to illustrate expected tuning behavior, but therein were included only points of particular values of current meant to favor
one of the peaks. Besides needing detailed information of wavelength and current-dependent refractive index shifts in the gratings themselves, for a self-consistent treatment we should also consider the interdependent changes of threshold carrier concentration, index, and modes which could occur in the active section because the peaks of the reflectivity product spectrum \( R_f R_b \) generally are of different amplitudes. Tuning from one peak to another corresponds to different threshold carrier concentrations, yielding different refractive indexes of the active region. This in turn leads to changing of the mode positions when tuning from one lasing frequency to another. This section estimates the importance of this effect. The gain in the active section can be treated as one comprised of real and imaginary parts:

\[
g = g' + ig'' = (\Gamma g_m - \alpha_\alpha) - i\beta_\alpha. \tag{24}
\]

To account in the phase condition (22) for the refractive index variation in the active section, a generalized linewidth enhancement factor can be employed, which is defined in this case as the ratio of the change in imaginary part of the gain at frequency \( \omega \), to the change of the real part of the net gain at the frequency \( \omega_L \) of the reflectivity product peak, since that is the gain which actually determines the threshold carrier concentration

\[
\zeta(\omega, \omega_L) = \frac{\Delta g''(\omega)}{\Delta g'(\omega_L)}. \tag{25}
\]

The propagation constant in the active region can be written in terms of its nominal value assuming an index of refraction based on the threshold carrier concentration at reference lasing frequency \( \omega_{LO} \), plus the term due to the change in index of refraction corresponding to the carrier concentration at the new threshold gain for the new peak wavelength

\[
\beta_\alpha(\omega, \omega_L) = \beta_\alpha(\omega, \omega_{LO}) + \zeta(\omega, \omega_{LO}) \partial g(\omega_{LO})
\]

\[
= \beta_\beta(\omega, \omega_{LO}) + \zeta(\omega, \omega_{LO}) [\Delta (\Gamma g_m - \alpha_\alpha)]. \tag{26}
\]

Substituting the threshold condition (23) into (26), we get

\[
\beta_\alpha - \beta_\beta(\omega, \omega_{LO}) + \zeta(\omega, \omega_{LO}) \left\{ \frac{1}{2L_a} \ln \left[ \frac{R_b(\omega_{LO})R_f(\omega_{LO})}{R_b(\omega_L)R_f(\omega_L)} \right] \right\}. \tag{27}
\]

This makes the phase condition for modes

\[
2\beta_\beta(\omega, \omega_{LO}) L_a + \zeta(\omega, \omega_{LO}) \ln \left[ \frac{R_b(\omega_{LO})R_f(\omega_{LO})}{R_b(\omega_L)R_f(\omega_L)} \right] + 2\beta_\beta(\omega, \omega_{LO}) + \phi + \phi_f = 2m\pi \tag{28}
\]

where \( \beta_\beta \) is written as a function of the current in the phase tuning region. Equation (28) differs from (22) by a term proportional to \( \zeta \). How important is it? To answer this, one could estimate lasing frequency shift due to these terms. Let’s roughly estimate the second term in the LHS of (28). From Fig. 11 with increasing tuning current, the maximum reflectivity product changes from \( \sim 0.3 \) to 0.2. Assuming \( \zeta \sim 4 \), the second term in (28) is approximately \( 4 \ln(0.3/0.2) \approx \pi/2 \) rad. This phase shift is four times smaller than the intermode phase shift \( 2\pi \), so the frequency shift we are looking for is a fraction of the mode spacing and can be taken care of by adjustment of the phase section tuning current. It is of importance, however, if the detailed tuning characteristics of a DBR laser are required.

VI. CONCLUSION

To summarize, we have proposed the concept and design of binary superimposed gratings for a DBR reflector which has a comb-like reflection spectrum that can be made to have nearly constant reflection peak heights suitable for tunable lasers. In comparing the alternative options in the DBR designs, we find that sampled gratings suffer from reflectance peaks which are inherently nonuniform in amplitude and whose amplitude decays with increasing total number of peaks faster than BSG’s and SSG’s, whereas BSG’s are at least as good as that of an SSG which represents the best DBR structure for tunable lasers to date, and resulted in demonstration of over 100-nm tunable lasers. However, the proposed BSG grading structure is much less demanding in implementation than the SSG, where the precision required is hardly within that of standard e-beam lithography. In addition, the BSG provides freedom in choosing the positions and amplitudes of reflectance peaks. This allows extension of the tuning range by accounting for nonuniform spectral shifts in tuning.

General design rules for the grating reflector pair were formulated, both for the individual grating characteristics to achieve a given reflection spectra and for requirements on both spectra for tunability and side-mode suppression. An analytical expression of the coupling strength of the BSG’s is proposed. Even for segment sizes of up to one half of the single grating period (i.e., 100 nm for typical DBR lasers), the coupling strength is reduced by only 35%. Calculation of the mode frequencies of a BSG-based tunable laser was presented. The system demand of high SMSR (>34 dB) was consequently shown to be readily met even under operation power of 3 mW for the lasers incorporating the BSG mirrors. A more self-consistent phase equation accounting for varying threshold carrier concentration in the active section was developed, and the resulting additional wavelength shifts expected were shown to be a fraction of the longitudinal mode spacing.

REFERENCES


